Exotic superconductivity of the twinning planes

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A phenomenological theory of 2D defects in superconductors is developed. The twinning planes on which a sudden phase change of π of the order parameter occurs and on which the vortex filaments terminate can achieve exotic superconducting states.

Khaĭkin and Khlyustikov[1] have reported observing a superconductivity localized at the twinning planes. This effect has recently acquired a special interest in view of the discovery of twinning domains. Buzdin and Bulaevskiĭ[2] proposed a simple phenomenological model for a localized superconductivity. The characteristic feature of their approach is the *a priori* assumption that the superconducting order parameter at the singular plane is continuous. In the present letter we develop a more systematic theory of the 2D defects in superconductors which is based on the Ginzburg-Landau theory.

In the case where a superconductor has an arbitrary planar defect, the total free energy is the sum of the bulk and surface parts:

$$\mathcal{F} = \int dV F_V + \int dS F_S \,, \tag{1}$$

where F_V is the typical Ginzburg-Landau energy

$$F_V = \frac{1}{4m} |(i\nabla + 2e\mathbf{A})\psi|^2 + a\tau|\psi|^2 + \frac{b}{2}|\psi|^4 ,$$

$\tau = (T - T_{cV})/T_{cV}$, T_{cV} is the critical bulk temperature, and a and b are the positive constants.

The surface integral can be evaluated from the $z = 0$ plane where the defect is situated. The free energy F_S of a unit surface, which depends locally on the order parameter ψ, can be expanded in powers of ψ. Since the $z = 0$ plane is a special surface, the function ψ, and hence the vector potential $\mathbf{A}$ in general, are, at $z = 0$, discontinuous functions of the coordinates. The vector potential can, however, always be rendered discontinuous by means of a discontinuous gauge transformation. Because of the condition under which the normal component of the magnetic field is continuous, we have $[H_z] = (\partial/\partial x)[A_y] - (\partial/\partial y)[A_x] = 0$, where $[f] \equiv f(z = +0) - f(z = -0)$. These expressions clearly imply that $[A_\alpha] = \partial f/\partial x_\alpha$, where f is a certain function x_α, $\alpha = 1,2$. We carry out the gauge transformation $\mathbf{A}' = \mathbf{A} + \nabla\chi$, in which the function χ behaves in the following way in the limit $z \to 0$: $\chi = z\chi_1(x_\alpha) + \chi_2(x_\alpha)$, where $[\chi_1] = -[A_z]$ and $[\chi_2] = -f$. We find $[A'] = 0$. Assuming that the vector potential is a continuous function, we will consider only the gauge transformations with continuous χ and $\partial\chi/\partial z$.

Because χ is continuous, we have four gauge-invariant quadratic combinations of the order parameter: $|\psi_+|^2$, $|\psi_-|^2$, $\psi_+^* \psi_-$, and $\psi_+\psi_-^*$, where $\psi_\pm = \psi(\pm 0)$. Taking into account the time-reversal symmetry $\psi \to \psi^*$ (we assume that the defect is nonmagnetic and that it is not characterized by a strictly 2D complex order parameter of any sort which is not related to the bulk order parameter ψ), we write the expansion

$$F_S = F_{S0} - A|\psi_+|^2 - B|\psi_-|^2 - C(\psi_+^*\psi_- + \psi_-^*\psi_+), \tag{2}$$

where F_{S0} is the free energy of the twin in the normal state, and A, B, and C are arbitrary positive or negative constants. Varying $\mathscr{F}$ in accordance with $\delta\psi$ in the volume and at the surface, we find the Ginzburg-Landau volume equation and the boundary conditions for $z = \pm 0$:

$$\xi_0\left[\left(\frac{\partial}{\partial z} - 2ieA_z\right)\psi\right]_{z=+0} = -\alpha\psi_+ - \gamma\psi_- , \tag{3}$$

$$\xi_0\left[\left(\frac{\partial}{\partial z} - 2ieA_z\right)\psi\right]_{z=-0} = \gamma\psi_+ + \beta\psi_- , \tag{4}$$

where $\xi_0^{-1} = 2\sqrt{ma}$, and the constants A, B, and C are written respectively in the form $a\xi_0\alpha$, $a\xi_0\beta$, and $a\xi_0\gamma$ with dimensionless α, β, and γ. If the Ginzburg-Landau theory is to be applicable in the entire temperature range of the localized superconductivity, we must stipulate, as can be seen from Eq. (6) derived below, that $|\alpha|$, $|\beta|$, and $|\gamma|$ be small. Using the standard expression for the bulk current and allowing for the continuity of its z component j_z, we find, for $z = 0$,

$$j_z = ie\sqrt{\frac{a}{m}}\,\chi(\psi_+^*\psi_- - \psi_+\psi_-^*). \tag{5}$$

Conditions of the type (3)–(5) were used by de Gennes[4] to describe the properties of the S–I–S and S–N–S contacts below T_{cV}.

Let us assume that in a certain temperature interval $0 < \tau < \tau_c$ there is a superconductivity which is localized near the defect. We evaluate τ_c in the absence of a magnetic field ($\mathbf{A} = 0$). The modulus ψ is small near τ_c and the Ginzburg-Landau equation can be linearized. We find $\psi(z) = \psi_{\pm} \exp[-(|z|/\xi_0)\sqrt{\tau}]$ for $z \gtrless 0$. Substituting (4) into (3), we find the following expression from the condition that the determinant of the system should vanish:

$$\tau_c^{1/2} = \frac{\alpha + \beta}{2} + \sqrt{\left(\frac{\alpha - \beta}{2}\right)^2 + \gamma^2}\,, \tag{6}$$

The fact that this expression must be positive is the only condition imposed on the three constants α, β, and γ, which accounts for the existence of a localized superconductivity at $T > T_{cV}$.

Substituting (6) into (3), we find

$$\psi_- = \frac{1}{\gamma}\left[\sqrt{\left(\frac{\beta - \alpha}{2}\right)^2 + \gamma^2} + \frac{\beta - \alpha}{2}\right]\psi_+ \,.$$

There are therefore two substantially different cases. If $\gamma > 0$, the order-parameter phase is continuous. If, on the other hand, $\gamma < 0$, the phase changes abruptly by an amount π when $z = 0$. These properties of the defect remain constant even at temperatures below T_{cV}. The phases in the bulk of the material lying on each side of the defect differ in this case by an amount π. The defect is therefore a boundary of unique superconducting domains.[1)]

In the absence of ψ, the twinning plane is always invariant under spatial transformation S (reflection in the $z = 0$ plane, rotation of C_2, or inversion), so that $\alpha = \beta$. At $\gamma < 0$ a state with an order parameter which is not invariant under spatial transformation is formed. It is, however, invariant under a combined $e^{i\pi} S$ transformation, which involves S and the multiplication of the order parameter by -1. This symmetry is similar to the symmetry of the exotic superconducting phases.[3]

A superconductor may directly reveal regions with phases that differ by an amount π. If the ends of a superconducting wire are attached to two points on the surface of the superconductor, a current will either flow in this wire or not, depending on whether the two points belong to domains of the same or different types.

Finally, we would like to make the following point. In the microscopic theory the twinning plane with $\gamma < 0$ should be characterized by the absence of ψ. The order-parameter phase at this plane is therefore not defined and the magnetic flux can easily enter the region lying near the point at which ψ vanishes. We thus clearly see that vortex filaments can terminate at these planes. The points at which they terminate are unique 2D magnetic charges. To identify the law describing their interaction, we introduce a 2D vector

$$b_\alpha(x_\alpha) = \int dz\, H_\alpha(z, x_\alpha)\,,$$

where the magnetic field is integrated over a narrow region near the twinning plane. If the values of $\mathbf{b}$ for the surface energy are small, we can use the expansion $F_S(\mathbf{b}) = F_S(0) + (\pi/\lambda)\mathbf{b}^2$, where λ is a constant with the dimensionality of length [in superconductors with a large Ginzburg-Landau parameter κ we have $\lambda = (4\pi)^2\delta$, where δ is the penetration depth]. The vector $\mathbf{b}(\mathbf{r})(\mathbf{r} = \{x_a\})$ satisfies (at $j_z = 0$) two-dimensional magnetostatic equations

$$\operatorname{div}\mathbf{b} = \sum_a \phi_a\,\delta(\mathbf{r} - \mathbf{r}_a), \quad \operatorname{curl}_z \mathbf{b} = 0,$$

where $\mathbf{r}_a$ are the coordinates of the points at which the vortices terminate, and $\phi_a = \pm\phi_0$; here ϕ_0 is a quantum of magnetic flux, and the sign depends on whether the vortex filament starts or ends at the plane. The energy of the interaction of two termination points is

$$U(r) = -\frac{\phi_1\phi_2}{\lambda}\ln\frac{r}{\delta}\,.$$

Since the attraction energy of the opposite termination points increases as $r \to \infty$ only logarithmically, the vortex filament which crosses a system of parallel twinning planes at an angle is unstable with respect to the division into sections which are nearly perpendicular to the planes. Each of these sections starts at the point the initial filament crosses one of the planes and ends at the neighboring plane. The energy of the filament increases because of the division into $\cos\theta$ (θ is the angle between the initial filament and the normal to the planes). Accordingly, the lower critical field should exhibit an anisotropy of the form $H_{c1}(\theta) = H_{c1}(0)\cos\theta$. The fact that the vortices can break up into short segments greatly reduces the effective pinning, with the exception of the case in which the vortices are parallel to the twinning planes.

To explain several anomalous properties of high-temperature superconductors, Deutscher and Müller[5] in a recently published paper assumed that the superconductivity near the twins is suppressed as the regions of the tunnel-junction type are formed. The theory developed in this study accounts for the Josephson tunneling along with the stimulation of an (exotic) superconductivity near a twin.

[1)]Asymmetric defects which are not invariant under time reversal always have a nonzero phase discontinuity.

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