Modulational instability of a plasma wave in a beat-wave accelerator

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The onset of a modulation instability of plasma waves excited at the beat frequency of two electromagnetic waves substantially lowers the efficiency of electron acceleration in a laser accelerator.

1. A possibility which is presently the subject of a lively discussion is the development of accelerators in which the energy of electrons is increased as a result of their resonant interaction with a plasma wave (or "Langmuir wave") excited by copropagating laser beams with nearly equal frequencies.[1] Interest in this suggestion has

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reached the point that the theoretical work has already been complemented with some first demonstration experiments.[2]

The efficiency of this acceleration method is determined by essentially two parameters: The maximum amplitude of the plasma wave, E_m, and its "lifetime" t_l. In the landmark papers on the subject, a limitation on the amplitude was linked with a breaking of the wave (the onset of a multistream motion of electrons oscillating in the wave field). In the present letter we examine the effect of a modulational instability on the excitation of a fast plasma wave. The results show that the onset of this instability can disrupt the fast plasma wave before electrons trapped by the wave are accelerated.

2. Let us examine the excitation of a fast plasma wave by two plane electromagnetic waves,

$$\mathbf{E}_1 = E_1 \mathbf{y}^0 \exp\left[\omega_1\left(t - \frac{x}{c}\right)\right], \qquad \mathbf{E}_2 = E_2 \mathbf{y}^0 \exp\left[\omega_2\left(t - \frac{x}{c}\right)\right], \tag{1}$$

which are propagating through a homogeneous plasma whose electron plasma frequency ω_0 is equal to the beat frequency: $\omega_0 = \omega_1 - \omega_2 \ll \omega_1, \omega_2$.

The electrostatic plasma wave $\vec{\mathscr{E}} = [\mathbf{E}(\mathbf{r},t)\exp(i\omega_0 t + \text{c.c.})]$, $|\partial \mathbf{E}/\partial t| \ll \omega_0 |\mathbf{E}|$, is excited by a nonlinear current at the frequency ω_0. The density of this current, $\mathbf{j}$, is given by[3] $\mathbf{j} = \mathbf{x}^0 ie\omega_0^2 E_1 E_2 \exp(-ikx)/\pi m(\omega_1 + \omega_2)^2 c$. Here $\kappa \simeq \omega_0/c$; c is the velocity of light; e and m are the charge and rest mass of the electron; $v_T = (T/m)^{1/2}$; and T is the plasma electron temperature. We restrict the analysis to the purely one-dimensional situation, in which the plasma nonlinearity is due exclusively to the dependence of the electron mass on the velocity at which the electrons oscillate in the wave field.[1)] In this case, in the approximation of a slightly relativistic situation, $E \ll E_r = \sqrt{2/3}\, m\omega_0 c/e$, the excitation of a fast plasma wave by the external field in (1) is described by the equation

$$i\frac{\partial e}{\partial \tau} - \frac{\partial^2 e}{\partial \xi^2} - |e|^2 e = d - i\hat{\gamma} e. \tag{2}$$

Equation (2) incorporates the dispersive and dissipative properties of the plasma. In it, we have introduced the dimensionless quantities

$$e(\xi, \tau) = E(x, t)/E_r, \qquad d = 4\pi |\mathbf{j}| / \omega_0 F_r, \tag{3}$$

$$\tau = \omega_0 t/2, \quad \xi = x\omega_0/\sqrt{3} v_T, \quad \hat{\gamma} = 2\hat{\nu}/\omega_0 .$$

The operator $\hat{\nu}$ describes the dissipation of the energy of the fast plasma wave. In the derivation of (2) it was assumed that the dimensionless wave number of the fast plasma wave is[2)] $\kappa' = \sqrt{3} v_T/c = 0$; i.e., the fast plasma wave in (2) corresponds to the zeroth spatial Fourier harmonic of the field, $e_0 = (1/2L)\int_{-L}^{L} e d\xi$. Equation (2) is a particular case of the mathematical models studied in Ref. 4, which describes the onset of a modulational instability and the excitation of a pronounced Langmuir turbulence in a plasma whose nonlinear properties are determined by the ponderomotive force. In particular, it was shown in Ref. 4 that the primary feature of the turbulence excited by a source with a given electric displacement "d" is the phase mechanism for the interac-

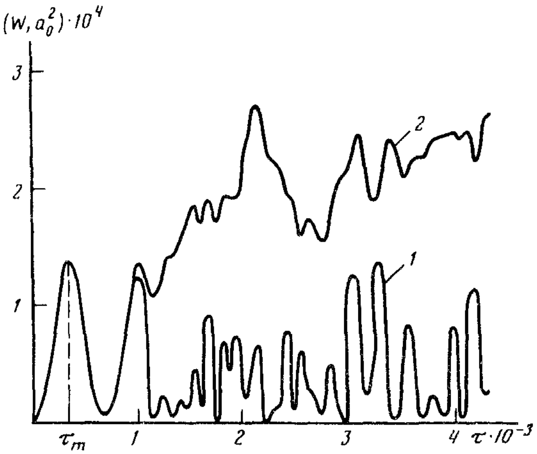

FIG. 1. Time evolution of the energy of a fast plasma wave (curve 1) and of the total energy of the plasma waves (curve 2), for $d = 4\times 10^{-4}$.

tion of the turbulence with the pump: The efficiency at which the source energy is "assimilated" is determined by the phase difference between the zeroth plasma-wave harmonic, $e_0 = a_0 \exp(i\varphi_0)$ (a_0 and φ_0 are real quantities), and the pump. For this reason, the modulation instability of a uniform field e_0 formed by a soliton spatial structure of plasma waves automatically gives rise to a nonlinear phase shift φ_0 and "uncouples" the plasma waves from the external source. As a result, the quantity a_0 decreases sharply when the modulational instability is "turned on."

The validity of the results given here for the field of a fast plasma wave excited by two given rf waves with nearly equal frequencies is confirmed by an examination of numerical solutions of Eq. (2). Figure 1 shows the time evolution of the energy of the fast plasma wave, a_0^2, and of the total energy in the turbulence,[3)] W. We see that a distinctive feature of model (2) is that the modulational instability occurs in a field e_0, which is oscillating in time (not monotonically increasing, as in Ref. 4). These oscillations, which are caused by a nonlinear ("relativistic") frequency shift of the fast plasma wave, can be studied analytically. For such a study, we should ignore the spatial dispersion ($\partial/\partial\xi = 0$) in (2). A corresponding analysis,[4)] carried out in Ref. 5, shows that the amplitude of the fast plasma wave reaches maximum values $a_0 = a_{0m}$ at the times $\tau = n\tau_m$ ($n = 1, 2, \ldots$);

$$a_{0m} = (4d)^{1/3}, \qquad \tau_m \simeq 4.8\,(4d)^{-2/3}.$$

3. It can be seen from Fig. 1 that the "lifetime" of the fast plasma wave, τ_l (the time that elapes before the onset of the modulational instability), is[5)] $\tau_l \simeq 3\tau_m$. Comparing τ_l with the duration of the acceleration of an electron by the fast plasma wave,[1] τ_a, given by $\tau_a = \pi\omega_1^2/\omega_0^2$, we find that in a sufficiently strong beat wave,

$$d \gtrsim d_a, \qquad d_a = (\omega_0/\omega_1)^3, \tag{4}$$

the destruction of the fast plasma wave by the modulational instability occurs before the electrons trapped by the fast plasma wave are accelerated.

Noting that a static magnetic field $\mathbf{B}_0 = B_0 y^0$, $B_0 \lesssim a_{0m} E_r \omega_0/\omega_1$, does not affect the dynamics of a fast plasma wave,[5] we can make a corresponding estimate for a surfatron.[1] Comparing the electron acceleration time in a surfatron,[6] $\tau_{as} \approx \omega_1^2/(\omega_0^2 a_{0m})$, with τ_l we find that the modulational instability prevents an electron from acquiring energy under the condition

$$d > d_s, \quad d_s = (4\omega_0/\omega_1)^6/4. \tag{5}$$

Using (3), we can rewrite conditions (4) and (5) as

$$q > q_a, \qquad \lambda^2 q_a = 10^{18}(\omega_0/\omega_1)^3, \tag{6}$$

$$q > q_s, \qquad \lambda^2 q_s = 10^{21}(\omega_0/\omega_1)^6. \tag{7}$$

Here q is the radiation intensity (in watts per square centimeter), and λ is the wavelength (in microns). For the experimental conditions of Ref. 2 [$\lambda = 10\ \mu$m, $\lambda_p = (2\pi/\kappa) = 100\ \mu$m], the "boundary" values of the radiation intensity in (6) and (7) are low: $q_a \simeq q_s \simeq 10^{13}$ W/cm^2. By way of comparison, we note that the flipping of a fast plasma wave with an amplitude of $E_{0m} = a_{0m} E_r$ would have occurred under these conditions only at flux densities $q \gtrsim 3 \times 10^{16}$ W/cm^2.

[1)]We are thinking of fairly strong waves $\mathbf{E}_1$ and $\mathbf{E}_2$, for which the relativistic nonlinearity "is triggered" (from the standpoint of the onset of the modulational instability) more rapidly than the ponderomotive force is. The corresponding condition on the pump level is $d > (m/16M)^{1/2}$, where M is the mass of the ion.

[2)]This limitation does not cause any qualitative changes in the course of the excitation of the fast plasma wave. If the pump level is not too low, $d > 16(v_T/c)^3$, this limitation will also leave the quantitative characteristics of the fast plasma wave unchanged.

[3)]In the numerical calculations it was assumed that the dissipation of the plasma-wave energy results from linear Landau damping.

[4)]Including numerical studies of the case of a relativistically intense fast plasma wave.[5]

[5)]a) It can be shown that in the absence of dissipation ($\hat{\gamma} = 0$) the ratio (τ_l/τ_m) does not depend on the pump level d. b) Strictly speaking, model (2) can be used to determine only the lifetime of the fast plasma wave. The nonlinear stage of the modulational instability may occur in a way different from that described by (2). Under the condition $d > (v_T/c)^{3/2}$, for example, the nonlinear stages should be accompanied by a flipping of the soliton structure that arises.

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