Formation of magnetic islands in a current sheet

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A regular reconnected magnetic structure of a current sheet, in the form of a chain of islands, is shown to form as a result of the spontaneous growth of random initial perturbations. The rate of this process is controlled by the plasma conductivity over broad ranges of the properties of the plasma.

1. The present experiments were carried out in the UN-Phénix θ-pinch device. The initial plasma (hydrogen, $n_0 = 7\times10^{11}$–2.5×10^{12} cm^{-3}, $T_{e0} = 1$–5 eV) is produced in a quasisteady magnetic field $B_0 = 310$–440 G, which is oriented along the axis of the device. The plasma is sustained by compression by the field of a shock turn of width $L = 30$ cm and diameter 18 cm, which surrounds the vacuum volume. The field of this turn increases sinusoidally over time with a half-period of 1.3 μs to a value $B_{\max} = 1400$ G. A cylindrical current sheet forms at the plasma boundary and converges on the axis of the device. At a time $\lesssim 80$ ns after the beginning of the process, the electron temperature rises above 100 eV because of Joule dissipation of the current in the anomalous resistance. Beyond this point, the plasma may be regarded as collisionless in the classical sense of the term.[1]

The magnetic structure of the sheet is studied with a movable system of six magnetic probes, which are separated along the radius and which measure the field component B_z, parallel to the axis of the device. The spatial resolution is 1 cm along the Z axis and 0.5 cm along the radius. The time resolution is ~ 10 ns, determined by the accuracy at which the time scales for the signals from the probes are synchronized. The measurements show that the magnetic structure of the sheet is axisymmetric, within the experimental errors; the lines of constant magnetic flux, $\Phi(r,z) = \int_0^r B_r(\rho,z)\rho d\rho$, then coincide with the lines of force of the field. The values of the field at each point are averaged over 3–5 shots.

2. Figure 1 illustrates the evolution of the magnetic structure. We see that magnetic islands (a,B) develop from initial perturbations of the magnetic field. In the course of the formation of the islands, there is an increase in the transverse field component $B_r(r_0,z)$ on the null line of the axial magnetic field $[B_z(r_0,z) = 0]$. This component is calculated from the expression $B_r(r_0 z) = -(1/r_0)\int_0^{r_0}(\partial B_z/\partial z)\rho d\rho$ (Fig. 1, b and d). The dynamics of the growth of the perturbations at various scales can be analyzed conveniently by expanding $B_r(r_0,z)$ in a Fourier series in spatial harmonics which are multiples of L:

$$B_r(t, r_0, z) = \frac{A_0(t)}{2} + \sum_{h=1}^{8} A_n(t)\cos\frac{2\pi h}{L}z - \varphi_n \tag{1}$$

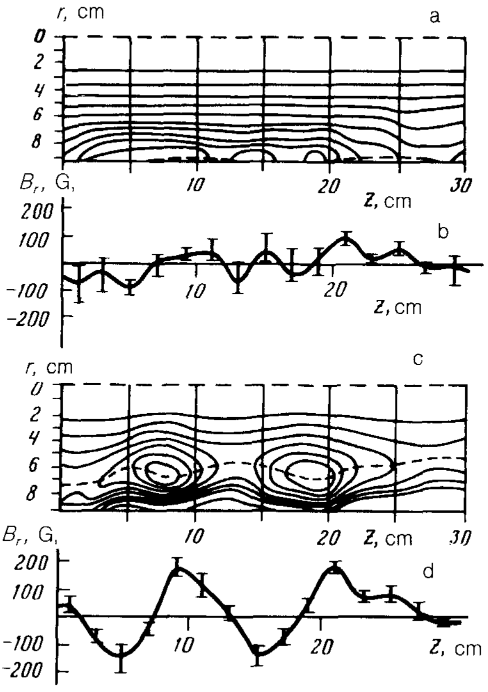

FIG. 1. Magnetic-flux contour maps at (a) 20 ns and (c) 120 ns; corresponding distributions $B_r(r_0,z)$ (b),(d) on the null line. The null line is the dashed line. The contour curves are drawn at a step of 1000 G·cm^2. They are bounded by the separatrix, with $\Phi = 0$. Here $z = 0.30$ cm is the boundary of the shock turn ($B_0 = 310$ G, $n_0 = 7\times10^{11}$ cm^{-3}).

The summation is carried out over the first eight harmonics, which provide a sufficiently accurate restoration of the original signal. Figure 2 shows the amplitudes and phases of the harmonics at the beginning and end of the island formation process; we see that the perturbations are detected over a broad range of scales: $\lambda_n = (L/n) = 4$–30 cm. The average amplitudes of the perturbations at the different scales are initially approximately the same; the scatter of amplitudes and positions of the perturbations along the Z axis (which determine the phases φ_n) in the different shots is large, so that the initial field perturbations are *random* in nature. We then observe a growth of the second through fifth harmonics; the scatter in the amplitudes and phases of these harmonics decreases sharply. The magnitude and position of the growing harmonics are thus fixed; i.e., they acquire a *regular* nature in the course of the formation of the islands. As can be seen in Fig. 2a, the following relation holds for them: $k_n\Delta < 1$ ($k_n = 2\pi/\lambda_n$, $\Delta \simeq 0.8$ cm is the half-width of the current sheet in the given case). This behavior is typical of specifically spontaneous reconnection.[2] When we therefore assume that the observed process is due to the onset of a tearing-mode instability of the current sheet, in accordance with ideas[3,4] regarding the nonlinear stage of the instability, we approximate the experimental time dependence of the amplitudes of the growing harmonics in expansion (1) by the expression

$$A_n(t) = A_n(t_0) + a_n(t - t_0)^2 \qquad (2)$$

The parameter a_n, averaged over all of the regimes studied (the regimes are defined by the initial parameter values n_0, B_0), is a measure of the average growth rate of the

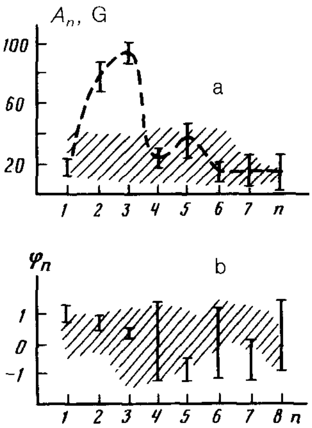

FIG. 2. Amplitudes (a) and phases (b) of the harmonics of a Fourier expansion of the magnetic perturbations. Hatching—scatter in the values at the initial time ($t = 20$ ns); vertical bars—scatter in a later stage ($t = 120$ ns).

corresponding harmonic. The dependence of $a_n^{(\mathrm{av})}$ on $k_n \Delta$ found in this manner confirms the conclusion reached above: that only the harmonics with $k_n \Delta < 1$ grow. This conclusion can therefore be regarded as statistically grounded.

3. The level of reconnection in a current sheet in which islands have formed can be characterized by the parameter $b = B_{rm}/B_z^{(b)}$, which reaches the value $b_{\max} \approx 0.2$–0.3 in the stage of reconnections which has been studied [B_{rm} is the maximum value, in magnitude, which $B_r(r_0,z)$ reaches in one island]. The time dependence of the parameter b is parabolic [see (2)], with a corresponding coefficient a. The values of a found for all the islands in a given regime from the experimental dependence $b(t)$ are then averaged, in order to characterize the average reconnection rate in this regime. Comparison of the results calculated for the average coefficient a for the various regimes shows that this coefficient depends on the plasma conductivity (Fig. 3). We calculated the value of σ from Ohm's law, $j = \sigma E$, using average values of j and E along the null line, which we found from the distribution of the magnetic field, $\mathbf{B}(r,z)$. We then also took an average of σ over the growth time interval, $b(t)$. Approximating the data in Fig. 3 by a power law $a(\sigma) \propto \sigma^{-\alpha}$ by the method of least squares, we find the value $\alpha = 0.45 \pm 0.16$. A functional dependence fairly close to that found here, $a(\sigma) \propto \sigma^{-2/3}$ is predicted by the theoretical model of a "semicollisional" mode of the nonlinear tearing-mode instability.[3,4]

4. Let us compare our results with those found by other authors. Figure 3 shows,

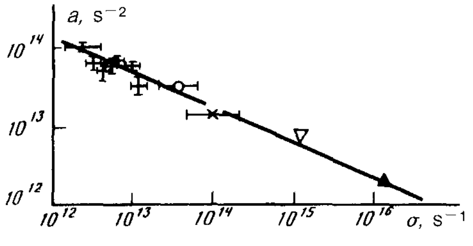

FIG. 3. Average reconnection-rate parameter as a function of the average plasma conductivity in the sheet. ⊢I⊣ —Results of the present study; ⊢O⊣—results of Ref. 6; ⊢×⊣ —Ref. 7; ▽—Ref. 8; ▲—Ref. 5. The straight line shows the function $a(\sigma) \propto \sigma^{-0.45}$.

along with the dependence $a(\sigma)$ found in the present experiments, the results of Refs. 5–8, where islands were also observed in a current sheet, under experimental conditions differing in the geometry of the sheet and the values of the plasma parameters. Under the assumption that the island-formation mechanism in these experiments is similar to that described above, we can also approximate the time dependence of the growing transverse field component $b(t)$ for those experiments by a function of the type in (2). For them, we assume $b_{\max}$ to be 0.2–0.3; this value is consistent with the experimental data reported in the other studies. It can be seen from this figure that the empirical dependence $a(\sigma) \propto \sigma^{0.45}$ found in the present study gives a satisfactory description of the experimental results over the conductivity interval $\sigma \simeq 10^{12}–10^{16}\ \mathrm{s}^{-1}$. We wish to stress that this dependence holds over a broad range of experimental conditions, including substantial differences in both the experimental geometry and the parameters of the plasma of the current sheet. This result shows that collisions in the plasma appear to play the role of a common, universal mechanism by which the reconnection process occurs, regardless of whether this reconnection is forced, as suggested in Ref. 6, or due to the spontaneous development of the instability of the sheet. In a dense cold plasma,[5] collisions are of a classical Coulomb nature, while for a collisionless plasma in the experiments described here the role of effective collisions is played by the scattering of electrons in the microfields of a *small-scale* (electrostatic) turbulence. The pressure of this turbulence is therefore fundamental to the change in the structure of the *large-scale* magnetic structure of the current sheet, controlling the rate of this process in a collisionless plasma.

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