Heavy fermions as a giant Migdal effect

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The interaction with a system of spins of *f*-atoms which are weakly coupled with each other is responsible for a pronounced dynamic renormalization of the spectrum of conduction electrons at a low temperature. This renormalization is analogous to a Migdal renormalization in an electron-phonon interaction. In a superconducting state, an anomalously large value of the coefficient κ arises as a result. In addition, there is a pronounced lowering of the sensitivity of the triplet state to scattering effects.

All of the heavy-fermion metals which have been identified so far have an *f*-atom—a lanthanide or an actinide—as one of their major constituents. There are several examples of systems of similar composition differing sharply in the extent to which heavy-fermion effects are manifested. The problem of the nature of these effects seems to be closely related to the question of what happens to *f*-electrons in a metallic matrix. On the other hand, it is extremely important to reach an understanding of which features in the state of the *f*-electrons in a metal favor the appearance of heavy fermions. Our purpose in this letter is to show that the basic set of low-temperature anomalies in heavy-fermion systems can be interpreted in a natural way by assuming that the *f*-electrons in these systems remain fairly well localized and that their ground term, which is formed if there is a strong spin-orbit coupling and if the crystalline surroundings have the appropriate symmetry, is nearly degenerate. If we introduce a generalized spin operator S to characterize this term, we can take the width (E_0) of the energy interval in which a substantial part of the spectral density of the time-dependent correlation function if the operators S at one *f*-atom e.g., $\langle TS_i(0)S_k(t)\rangle$, as a measure of the smearing of the term. The width E_0 determines that region of low temperatures in which the heavy-fermion effects unfold. In many cases, this width is only a few degrees along the temperature scale. The conduction electrons are bound to the magnetic terms by the short-range potential V which depends on the spin. This coupling also leads to a pronounced renormalization of the electron spectrum in a small neighborhood, $\sim E_0$, of the Fermi surface. Here there is a significant analogy with the Migdal effect.[1] Migdal showed that the eigenenergy $\Sigma(\epsilon,\mathbf{p})$ in the Green's function of the electrons, $G=[\zeta_\mathbf{p}-\epsilon-\Sigma]^{-1}$, $\zeta_\mathbf{p}=v(p-p_0)$, which is associated with the electron-phonon interaction, has the property $\lambda_\epsilon=[\partial\Sigma(\epsilon,p_F)/\partial\epsilon]\sim 1$ in the limit $\epsilon\to 0$, while we have $\lambda_p=v^{-1}[\partial\Sigma(0,p)/\partial p]\sim(\hbar\omega_D/E_F)\ll 1$. The renormalization of the spectrum that arises is therefore dynamic: $v^*=v(1+\lambda_\epsilon)^{-1}$. It exists at $T<\hbar\omega_D$ in a layer $\sim\hbar\omega_D$ near the Fermi level (ω_D is the Debye frequency of the phonons). The reason is that the effective interaction between electrons through the phonon field is a short-range but highly retarded interaction. Simplifying the matter slightly, we could say that electrons which interact through the same ion approach it

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with a time interval ω_D^{-1}, and at each given instant they are at a significant distance from each other. It is thus specifically the electron-phonon interaction that competes so successfully with the Coulomb repulsion upon the onset of superconductivity. In the case in which we are interested here, the retardation effect is even more pronounced, since it is determined by an extremely small quantity. Accordingly, there must be a strong dynamic renormalization of the spectrum. The probability for a dynamic origin of the large effective mass of heavy fermions seems to have been pointed out originally by Varma[2]: λ_ϵ and λ_p appear differently in the expressions for the thermodynamic and kinetic quantities. Varma believed that the data available constituted evidence in favor of λ_ϵ.

Let us examine some effects which arise in second order in the interaction (V) of the electrons with each of the f-atoms. For clarity, we will use a simple model in which two-level centers with a splitting energy E play the role of f-atoms. To simulate the smearing of the f-term mentioned above, we assume that there is a distribution in E in an interval $\sim E_0$ with a normalized density $w(E)$. In the thermodynamic technique $[\epsilon = i\epsilon_n, \epsilon_n = (2n+1)\pi T]$ the expression for Σ is ($\hbar = 1$).

$$\Sigma(i\epsilon_n, \mathbf{p}) = n_s T \sum_{n'} \int \frac{d^3\mathbf{p}'}{(2\pi)^3} |V_{\mathbf{p},\mathbf{p}'}|^2 \int dE w(E) \frac{2E}{(\epsilon_n - \epsilon_{n'})^2 + E^2} \operatorname{th} \frac{E}{2T} G(i\epsilon_{n'}, \mathbf{p}'), \tag{1}$$

where n_s is the density of f-atoms. The part of Σ which is odd in ϵ_n, i.e., $f(\epsilon_n)$, is determined by a region of $\mathbf{p}'$ close to the Fermi surface, and we can transform to an integration over this surface and over $\zeta_{p'}$. Here we report the results for two limiting cases. 1) $T \gg E_0$. The dominant contribution comes from $n' = n$:

$$f(\epsilon_n) \simeq i \operatorname{sign} \epsilon_n \frac{1}{2\tau_s}, \qquad \frac{1}{\tau_s} = \frac{n_s}{4\pi^2} \int \frac{dS}{|v|} |V_{p,p'}|^2 \tag{2}$$

We see that here there is only a damping, which does not depend on T. 2) $T = 0$. Transforming to an integral over ϵ', we find

$$f(\epsilon_n) \simeq \lambda_\epsilon i\epsilon_n, \qquad \lambda_\epsilon = \frac{1}{\pi\tau_s} \int dE \frac{w(E)}{E} \gtrsim \frac{1}{\pi\tau_s E_0} \gg 1. \tag{3}$$

The slight damping here depends on the functional dependence $w(E)$. The behavior of τ_s^{-1} and λ_ϵ as functions of T is shown qualitatively in Fig. 1. The quantity λ_p is

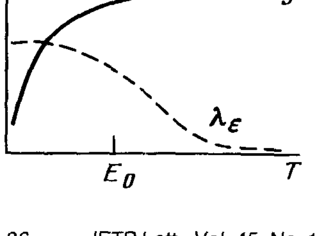

FIG. 1.

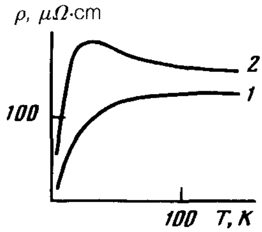

FIG. 2.

determined by the part of Σ which is even in ϵ_n, and it turns out to be small, as expected: $\lambda_p \sim (\tau_s E_F)^{-1} \ll 1$.

Information on τ_s can be extracted from data on the resistivity[3] ρ. Figure 2 shows two typical schematic curves of (T); the typical values of the quantities are indicated. We might expect that the value of ρ on the plateau of curve 1 would be determined by scattering by essentially free spins (2). Its value corresponds to a mean free path $l \sim 10^{-6}$ cm; here we have $\tau_s \sim 10^{-14}$ s. In view of what was said above about the value of E_0 [this quantity can also be estimated from the decay interval of $\rho(T)$], we can find the rough estimate $\lambda_\epsilon \sim 10^2$. A value of λ_ϵ on the same order of magnitude follows from the observed linear term in the heat capacity, the expression for which contains[4] $1 + \lambda_\epsilon$.

The expression derived here can be recommended for a quantitative comparison with experiment if E_0 is dominated by the interaction between terms of different f-atoms. This interaction is relatively insensitive at short range to the properties of electrons near the Fermi level, which can therefore be calculated with the help of an independently determined spin correlation function. The relatively slight manifestations of the Kondo effect in this case (e.g., curve 2 in Fig. 2) can be taken into account in third order in V. The case of a pronounced Kondo effect requires a self-consistent description of the f-terms and the conduction electrons.

These problems do not bear on the possibility of a semiphenomenological description of the electron properties $T < E_0$ in terms of a dynamic renormalization of the spectrum. This assertion also applies to the theory of superconductivity in heavy-fermion systems. Here we could use essentially the same formalism as the case of an electron-phonon interaction.[5] Here are some of the consequences of this approach: 1) The equation for the order parameter contains the coupling constant with a weight $(1 + \lambda_\epsilon)^{-1}$. Accordingly, only an interaction that leads to a large value of λ_ϵ is effective; all competing interactions are suppressed. 2) The coefficient in the theory of Ginzburg and Landau is $\kappa \sim \lambda_\epsilon^{3/2}$. This result leads to a very large value of H_{c2} and to an extremely narrow region for the Meissner effect. These conclusions are in accordance with the observed properties. 3) The relaxation time τ, which determines the resistance, is part of the equation for the gap with a weight $(1 + \lambda_\epsilon)\tau$. The effect is to greatly reduce the sensitivity of the triplet state to various scattering effects.

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[1]A. B. Migdal, Zh. Eksp. Teor. Fiz. **34**, 1438 (1958) [Sov. Phys. JETP **7**, 996 (1958)].
[2]C. M. Varma, Phys. Rev. Lett. **55**, 2723 (1985).
[3]G. R. Stewart, Rev. Mod. Phys. **56**, 775 (1984).
[4]G. M. Éliashberg, Zh. Eksp. Teor. Fiz. **43**, 1105 (1962) [Sov. Phys. JETP **16**, 780 (1962)].
[5]G. M. Éliashberg, Zh. Eksp. Teor. Fiz. **38**, 966 (1960) [Sov. Phys. JETP **11**, 696 (1960)]; **39**, 1437 (1960) [**12**, 1000 (1961)].

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