Superconducting transition temperature in amorphous films

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The results of experimental studies of the suppression of a superconducting transition in homogeneous amorphous films are analyzed. To correctly describe this effect, it is necessary to use, even at $t \sim 0.01$, the renormalization-group equations in addition to a first-order correction in $t = (e^2/2\pi^2\hbar)R_\square$. The plot of the transition temperature versus $R_\square$ constructed from these equations describes well the available experimental data.

Al'tshuler *et al.*[1] have established that Coulomb interaction of electrons, when combined with the scattering by impurities, gives rise to various anomalies (see the review by Lee and Ramakrishnan[2]). Specifically, it was predicted in Refs. 3 and 4 that a presence of disorder lowers the temperature T_c of the superconducting transition of films in the following way:

$$\ln \frac{T_c}{T_{c0}} \approx -\frac{1}{3} g_1 \frac{e^2}{2\pi^2\hbar} R_\square \left(\ln \frac{1}{T_c \tau}\right)^3, \tag{1}$$

where T_{c0} is the transition temperature in a bulk sample, $R_\square$ is the surface resistivity of the film, τ is the transit time, and g_1 is a constant which describes the screened Coulomb interaction. To eliminate the influence of secondary phenomena in the analysis of this effect, the bulk properties of the electron gas in the films with different values of $R_\square$ must be kept constant. In homogeneous films of different thicknesses but same composition, the bare values of the Fermi-liquid electron parameters (state density, coupling constants, etc.) may be assumed constant.[1)] The measurement of T_c as a function of $R_\square$ for a series of these films therefore makes it possible to directly determine the effect of the interaction and scattering of electrons[1,2] on the superconductivity of disordered two-dimensional systems. In an effort to determine this effect, measurements with amorphous films were carried out: W–Re (Ref. 6), Mo–Ge (Ref. 7), and Nb_3Ge (Ref. 8). Two circumstances dictated the choice of these materials. First, in these materials $\epsilon_F \tau$ is ~ 1 and hence $\ln 1/T_{c0}\tau$ is $\sim$5–10. Although many factors affect T_c of the films (see the review by Komnik[9]), the mechanism which we are analyzing here must therefore be the dominant mechanism because of the large value of $(\ln 1/T_c\tau)^3$. Secondly, to separate this effect from the "geometric" effects such as percolation or granulation, the films of different thicknesses must be homogeneous. Both the fabrication technology and the choice of components of the composition are important factors in providing a good homogeneity. In particular, the Mo–Ge films were fabricated in such a way that they remained homogeneous to thicknesses of ~ 10 Å.

It was found in Refs. 6–8 that the transition temperature in such films decreases by one-half even at $R_\square \sim 0.5$ kΩ in comparison with the bulk value of T_{c0}, and the superconducting transition in exceptionally thin but still homogeneous films was found in Ref. 7 to be suppressed completely at $R_\square \sim 2$–3 kΩ. In the experiments discussed here $T_c(R_\square)$ initially decreases with increasing $R_\square$ in a linear manner, consistent with the prediction of Eq. (1). At $R_\square > 0.5$ kΩ, however, T_c decreases more slowly. It was indicated in Ref. 7 that Eq. (1) should be modified in some manner.

Equation (1) was obtained by subtracting the first-order error in the parameter, $t = (e^2/2\pi^2\hbar)R_\square$. We will show below that at $R_\square > 0.5$ kΩ $T_c(R_\square)$ can no longer be determined by simply using the first correction (although the parameter t is still extremely small, $t \sim 0.01$) and that a renormalization-group equation should be used in this case. The renormalization-group equation for the coupling constant in the Cooper channel in the first order in t can be written as[10,11]

$$\frac{d\Gamma_c}{dy} = \frac{1}{2}t(Z + m\Gamma_2) - \frac{\Gamma_c^2}{Z}, \qquad (2)$$

where $y = \ln 1/\omega\tau$, Γ_c and Γ_2 are the amplitudes for the interaction of electrons in the Cooper channel and in the spin-density channel, Z describes the interaction of long-wave fluctuations of the electron density and is also a parameter which describes the renormalization of the coefficient at the diffusion and cooperon frequency in the propagator,[10] and m is the number of effective triplet components of the cooperon: $m = 3$ if there is no spin-orbit scattering and $m = 0^{2)}$ if $T_c\tau_{s0} < 1$ (τ_{s0} is the time of the spin relaxation due to spin-orbit coupling). The first term in (2) arises due to the renormalization of the amplitude Γ_c by the impurity-scattered charge-density fluctuations and electron-gas-spin fluctuations. As long as the reduction of the transition temperature is relatively small, the value of $T_c(R_\square)$ found from Eq. (2) will correspond to Eq. (1), in which the parameter g_1 should be replaced by $1/2(Z + m\Gamma_2)$.

Specific data on the value of Γ_2 and on the strength of spin-orbit coupling for the systems under discussion were not available to this author. Here, however, we have the following in mind. In the bulk sample the inclination of the critical field dH_{c2}/dT makes it possible to measure the diffusion coefficient. In view of this circumstance, the parameter g_1 was estimated in Refs. 6–8 by analyzing the experimental data on $T_c(R_\square)$ on the basis of Eq. (1). The value of this parameter turned out to be fairly close to 0.5 in all three cases. Since $1/2\ (Z + m\Gamma_2)$ is the parameter g_1 in Eq. (2), and since the bare value of Z is 1, the fact that g_1 is close to 0.5 means that $m\Gamma_2$ is small. For definiteness, we assume that $m = 0$, since the presence of reasonably heavy elements suggests that the spin-orbit coupling is appreciable. In first order in t, the renormalization-group equations, which describe the behavior of the system, can then be written[11]

$$\frac{d\Gamma_c}{dy} = \frac{1}{2}tZ - \frac{\Gamma_c^2}{Z}; \quad \frac{dZ}{dy} = t\left(-\frac{Z}{2} + \Gamma_c\right), \qquad (3)$$

$$\frac{dt}{dy} = t^2\left(\frac{1}{2} - \frac{\Gamma_c}{Z}\right). \qquad (4)$$

The superconducting transition temperature T_c can be determined without regard for the renormalization of t, since Eq. (4) begins to take effect only at $T_c/T_{c0} < 0.1$. Integrating Eq. (3), we then find

$$T_c/T_{c0} = \exp(-1/\gamma) \left\{ \left[1 + \frac{(t/2)^{1/2}}{\gamma - t/4} \right] \Big/ \left[1 - \frac{(t/2)^{1/2}}{\gamma - t/4} \right] \right\}^{1/\sqrt{2t}}, \tag{5}$$

where $\gamma = 1/\ln T_{c0}\tau (\gamma < 0)$. It follows from Eq. (5) that there are two scale dimensions of t which describe the decrease of the superconducting transition temperature as the resistivity $R_\square$ is increased: According to (1), T_c initially decreases appreciably at $t \sim |\gamma|^3$ and then slows down; the superconductivity vanishes at larger $t (t \sim \gamma^2)$. A simple allowance for the first-order correction in t is therefore not enough for a correct description of the suppression of superconductivity, despite the fact that $t \sim 0.01$. Accordingly, the retrogressive behavior of $T_c(R_\square)$ found from Eq. (1) in fact does not exist.

In the analysis of the data of Ref. 7 on the basis of Eq. (5) we assumed that $\ln 1/T_{c0}\tau = 8.2$, which is reasonably close to the value used in Ref. 7. Figure 1 shows that Eq. (5) describes very well the suppression of the superconductivity observed in amorphous homogeneous $Mo_{79}Ge_{21}$ films. It should be noted, however, that the validity of Eq. (5) is based on the presence of a special circumstance. The point we wish to make here is that $\tau^{-1} > \omega_D$ in the compounds we are discussing here (ω_D is the Debye frequency), so that the equation which describes the amplitude Γ_c in each region, $\tau^{-1} > \omega > \omega_D$ and $\omega_D > \omega$, should be integrated separately. For reasonable parameter values, however, the variation in the value of Γ_c, which occurs as a result of interference of the Coulomb interaction and $2D$ electron scattering, is nearly independent of the value of Γ_c in the region $\tau^{-1} > \omega > \omega_D$, since in Eq. (2) the term which contains t does not depend on Γ_c. For this reason, in the derivation of Eq. (5) we did not consider separately the frequencies above and below ω_D, although, strictly speaking, this was not always the case.

We note in conclusion that we have examined here a case of interest in the theory of disordered systems, in which, on the one hand, the renormalization-group equations

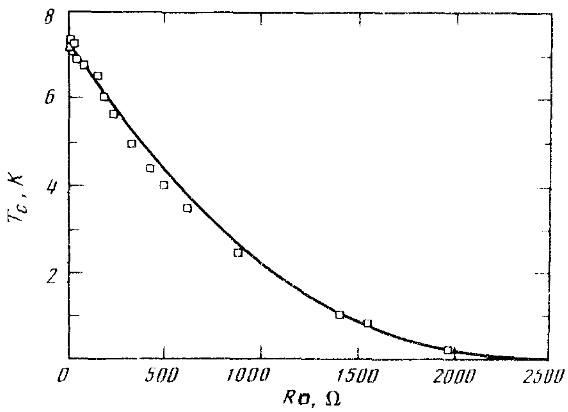

FIG. 1. Suppression of superconductivity in amorphous $Mo_{79}Ge_{21}$ films. □—Data of Ref. 7; solid line—result of analysis based on Eq. (5).

must be used to describe the effect and, on the other, the conditions for the applicability of these equations in lowest order in t are satisfied. In the films which we have considered the superconductivity is suppressed due to the joint action of the electron-density fluctuations and the scattering by impurities. This mechanism (plus a similar action of the spin-density fluctuations) is quite probably responsible for the fact that the superconductivity of homogeneous $3D$ materials vanishes on the metallic side of the metal-insulator transition.

[1)]We have disregarded here the effect on T_c of the transverse quantization which was discussed in Ref. 5, for example, and which seems to be unimportant for the compounds here. An expression for the correction to T_c due to the electromagnetic-field fluctuations, which is identical to Eq. (1) (if we set $g_1 = 1/2$) but with an opposite sign, was obtained in Ref. 5.

[2)]Equation (2) with $m = 0$ was used in the analysis of the superconductivity near the metal-insulator transition.[12]

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