Metastable vortex in superfluid ^{3}He-*B*

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A new vortex solution of the Ginzburg-Landau equations is derived. This new solution describes a single-quantum vortex with a fourfold symmetry axis. This vortex may explain an unusual metastable state which is observed in hysteretic processes. The physical properties of the vortex are discussed.

Nuclear-magnetic-resonance experiments on rotating ^{3}He-*B* have revealed the existence of at least two stable vortices which have a single quantum of circulation of the superfluid velocity and which differ in core structure.[1,2] Numerical analysis of the Ginzburg-Landau functional near T_c has so far demonstrated the existence of two stable vortices: an axisymmetric one[3,4] [of magnetic class $C_{\infty v}(C_\infty)$] and an axially asymmetric one[5,4,6] [of magnetic class $C_{2v}(C_2)$], or *v*-vortices. For each, parity is violated in the core.

These two vortices may correspond to vortices which are observable experimentally: The axially asymmetric *v*-vortex may correspond to a vortex at low pressures, while the axisymmetric vortex may correspond to one at high pressures. In order to make an identification of this sort, however, we still need (first) experimental proof that the line of the observed first-order phase transition between vortices goes onto the T_c line. Second, it may turn out that second-order phase transitions occur far from T_c but have not yet been observed because of the continuity of the measured physical quantities, for which the symmetry of the vortices found theoretically near T_c changes. Consequently, direct proof would consist of the observation of physical consequences of the breaking of various symmetries in the core.[7]

Furthermore, the unusual behavior of the hysteresis loop (see Fig. 5 in Ref. 8) suggests that yet another vortex—a metastable one—may exist. In order to determine which symmetry class might correspond to this hypothetical third vortex, we have sought possible metastable states near T_c. For this purpose we minimized the Ginzburg-Landau functional in classes of functions of various symmetries, making use of the circumstance that by virtue of the symmetry the minimum which is found is always a solution of both the Ginzburg-Landau equations and the Gar'kov equations, although it does not necessarily correspond to an absolute minimum or even a local minimum on the set of all functions.

We found that, along with the maximally symmetric *o*-vortex[9,3] [magnetic class $D_{\infty h}(C_{\infty h})$], which seems to always be unstable, there is at least one more—a fourth—solution, which belongs to class $D_{2d}(S_4)$. The new vortex is more preferable than the *o*-vortex from the energy standpoint; it is stable with respect to a transition to an axisymmetric *v*-vortex, while it is unstable with respect to a transition to an axially asymmetric *v*-vortex. Table I is a subordination diagram of the various symmetry

TABLE I. Symmetry breaking in quantized vortices in ^{3}He-B

$D_{\infty h}(C_{\infty h})$ o-vortex (vortex with maximal symmetry),[9,3]	→	$C_{\infty v}(C_{\infty})$ axisymmetric v-vortex[3,4]
⇣		⇣
$D_{2d}(S_4)$ vortex with square core	→	$C_{2v}(C_2)$ axially asymmetric v-vortex[4–6]
⇓		⇓
$D_2(C_2)$ spiral w-vortex[7]	→	C_2 spiral uvw-vortex

The different types of arrows show different types of symmetry breaking

→ Breaking of a discrete symmetry, in the course of which a spontaneous electric polarization arises in the core of the vortex along the axis of the vortex, and a spontaneous spin flux arises.

– – → Breaking of axial symmetry.

⇒ Complete breaking of spatial parity, which gives rise to a cholesteric spiral in the vortex core and to a spontaneous mass flux along the axis.

The four upper vortices were found as solutions of the Ginzburg-Landau equations near T_c; the existence of the last two vortices has not yet been checked in the Ginzburg-Landau region.

classes of the vortices. The different types of arrows indicate different types of symmetry breakings in transitions from one core structure to another.

The new vortex has the following order-parameter structure, which corresponds to class $D_{2d}(S_4)$. Among the parameters $C_{\mu\nu}(r)$ in the expansion of the amplitudes $a_{\mu\nu}$ for Cooper pairing with spin projection $\mu = 0, +1, -1$ and with orbital-angular-momentum projection $\nu = 0, +1, -1$ in the harmonics of Q,

$$a_{\mu\nu}(r,\varphi) = \exp[i(1-\mu-\nu)\varphi] \sum_Q C_{\mu\nu}(r, Q) e^{iQ\varphi}$$

(r is the distance from the vortex axis, and φ is the azimuthal angle), the following are nonzero and real; $C_{\mu\nu}(Q=4k)$ with $\mu+\nu$ even and $C_{\mu\nu}(Q=4k+2)$ with $\mu+\nu$ odd (k is an integer). In these calculations we retained only three harmonics: $Q=0$, $+2$, -2 (Fig. 1). This approach does not alter the symmetry or the topology, so that it does not alter the conclusions.

The symmetry elements of this vortex are formed from the combined symmetries PC_4 and TU_2. The quantity $\Sigma_{\mu\nu} a_{\mu\nu} a^*_{\mu\nu}$ has a fourfold symmetry axis; i.e., the cross section of the vortex has the symmetry of a square (Fig. 2). The properties of the vortex which follows from the symmetry, and which can be used to distinguish it experimentally from other (stable) vortices, are as follows: As in the case of o-vortices, there are no spontaneous spin or mass fluxes or spontaneous electric polarization. In the tensor of the vortical magnetic anisotropy,[5] the only nonvanishing component is

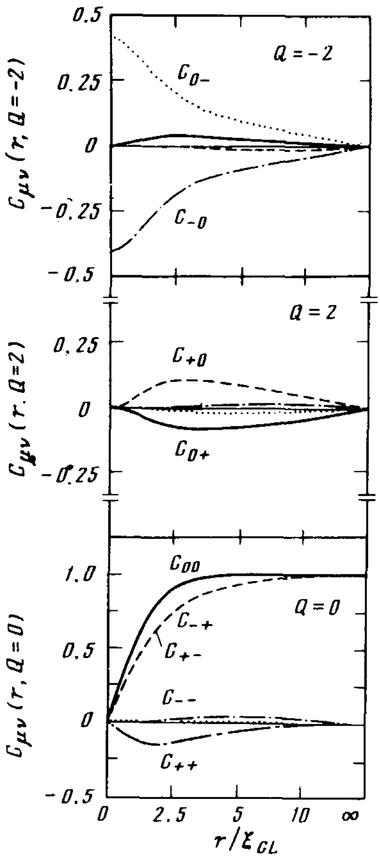

FIG. 1. The amplitudes ($C_{\mu\nu}$) for Cooper pairing in the new (metastable) vortex, with symmetry D_{2d} (S_4), versus the distance from the vortex axis, r. The nonzero amplitudes are those with even $\mu+\nu$ for the zeroth harmonic ($Q=0$) and with odd $\mu+\nu$ for the second harmonics ($Q=+2,\ -2$). A vortex may correspond to either a saddle point of the energy functional or to a local minimum.

λ_{11}, since there is no anisotropy vector in the cross-sectional plane of the vortex. In contrast with the o-vortex, on the other hand, the superfluidity in the core is not disrupted by the formation of boojums on the Fermi surface. The topology of the boojums on the Fermi surface is the same as that of the axisymmetric v-vortex and therefore different from the topology of the axially asymmetric v-vortex, which consists of a bound pair of vortices with a half-integer circulation.[6] Consequently, a topo-

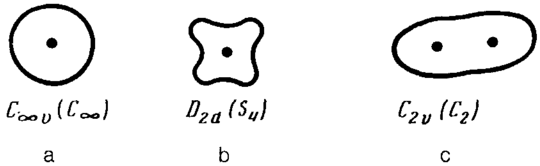

FIG. 2. Cross-sectional shapes of cores of various vortices. Within a core, boojums appear on the Fermi surface (zeroes of the gap in the excitation spectrum[6]). At the points, the l-vectors of the boojums are parallel to the vortex axis. a—v-vortex with "circular" core; b—new vortex with "square" core; c—v-vortex with "double" core (a pair of half-quantum vortices).

logical barrier which separates the vortex $D_{2d}(S_4)$ from vortex $C_{2v}(C_2)$ can appear far from T_c, so that the new vortex may be locally or even absolutely stable.

We note in conclusion that there has so far been no study of the stability of the solutions for the v-vortices with respect to the formation of a spiral cholesteric texture in the core of the vortex, which should arise in the case of axially asymmetry vortices upon a complete breaking of parity (Table I). This study will require minimizing the Ginzburg-Landau functional in all three spatial dimensions, since the length of the vortex itself becomes a spiral.

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