Superconductivity of quasiperiodic layered structures

A. Yu. Kitaev and L. S. Levitov
L. D. Landau Institute of Theoretical Physics, Academy of Sciences of the USSR

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The critical behavior of thermodynamic quantities at the point of the superconducting transition in a quasiperiodic layered structure is not the usual behavior. The critical indices are calculated in the Ginzburg-Landau approximation.

1. An unusual temperature dependence was recently observed for the upper critical field H_{c2} in an *SNSNS* system of V and Mo layers.[1] The thickness of the layers of the normal metal, d, was constant, while the superconducting layers had two different thicknesses, d_A and d_B, alternating in accordance with the Fibonacci law *ABAABABAABAAB*... . Near the transition point in a zero field, H_{c2} was found to be a nonlinear function of $\tau[\tau = (T_c - T)/T_c]$. Measurements of the coherence length ξ_0 revealed $d \ll \xi_0$ (strong coupling). If the results of Ref. 1 are plotted in logarithmic scale, a straight line with a slope of 0.74 results. This critical index lies between 1.0 (which corresponds to a periodic system of strongly coupled layers) and 0.5 (a separate layer) (Fig. 1).

Since the size of the nucleating region of the superconducting phase in a weak field is much greater than the lattice constant, the reason for the appearance of an anomalous critical index is a quasiperiodic nature of the structure. Other thermodynamic quantities—the order parameter Δ, the heat capacity C, the correlation length ξ, the London length λ, and the lower critical field H_{c1}—should also be power functions of τ with unusual exponents.

2. Let us consider the problem of calculating the critical indices in the Ginzburg-Landau approximation. We assume that the layered structure is described by a piecewise constant function $U(x)$, which takes on the constant values U_s and $-U_N$ within each layer ($U_S > 0$, $U_N > 0$—a proximity effect). The Ginzburg-Landau functional has the standard form:

$$F(\Delta(x)) = \left[\frac{1}{2}|\vec{\partial}\Delta|^2 + U(x)|\Delta|^2\right] - \tau|\Delta|^2 + |\Delta|^4 \qquad (1)$$

(the magnetic field H is incorporated in the gradient term). Near the transition point, the properties h, $|\Delta|$, and τ are small, so that the order parameter is a linear combination of the eigenfunctions of the lower part of the spectrum of the operator in square brackets. The properties of operators of this type were studied in detail in Ref. 2, where it was shown that all the edges of allowed bands are scaling points of the spectrum. This result means that if we place the origin of the energy scale at the lower edge of the spectrum the fraction of states with energies below ϵ will be related to ϵ in the following way:

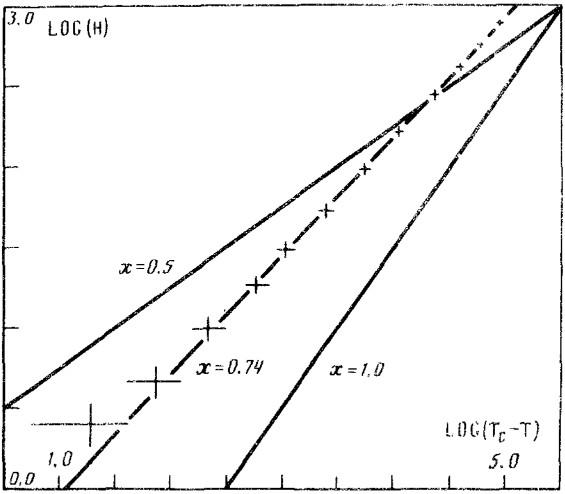

FIG. 1. Data of Ref. 1 in logarithmic scale. The slope x corresponds to the exponent in the law $H_{c2} \sim \tau^x$.

$$\epsilon = n^{2\gamma} F(\log_\Phi n), \qquad \Phi = \left(\frac{1}{2}(\sqrt{5}+1)\right)^2 . \tag{2}$$

Here Φ is the square of the golden section, and F is a bounded periodic function with a period of 1. The index γ is not universal; it is a function of the invariant J:

$$\gamma = \frac{1}{2}\log_\Phi\left[\frac{1}{2}\left((8t-1) + ((8t-1)^2 - 4)^{1/2}\right)\right], \qquad 4t^2 - 3t = J. \tag{3}$$

For our model, the invariant is given by an expression of the type

$$J = 1 + \frac{(U_S + U_N)^2}{4U_S U_N} \sinh^2(\kappa d)\sin^2(k(d_A - d_B)), \quad k = \frac{(2U_S)^{1/2}}{\xi_0}, \quad \kappa = \frac{(2U_N)^{1/2}}{\xi_0} \tag{4}$$

Another quantity of importance here is the critical index of the eigenfunction with the minimum energy:

$$\delta = \lim_{L\to\infty} \log_L\left[\left(\int_0^L \psi^2\right)^2 \Big/ \left(\int_0^L \psi^4\right)\right]. \tag{5}$$

As J varies from 1 to ∞, δ varies from 1 to $(\log_2 \Phi)^{-1} = 0.72...$. The index δ can be calculated in the following way. It was shown in Ref. 2 that the bottom of a band corresponds to a sequence of transfer matrices of the type $A, B, S^{-1}AS, BS, S^{-2}As^2, \ldots$ (the sequence of traces has a period of 2). The matrices A, B, and S satisfy the equations $AB = S^{-1}AS$, $BS^{-1}AS = S^{-1}BS$ and the condition $\mathrm{tr}(ABA^{-1}B^{-1}) = 2J$. The solution of these equations gives us A, B, and S as functions of J. We now need to determine that vector which, when acted upon by the transfer matrices, gives us the

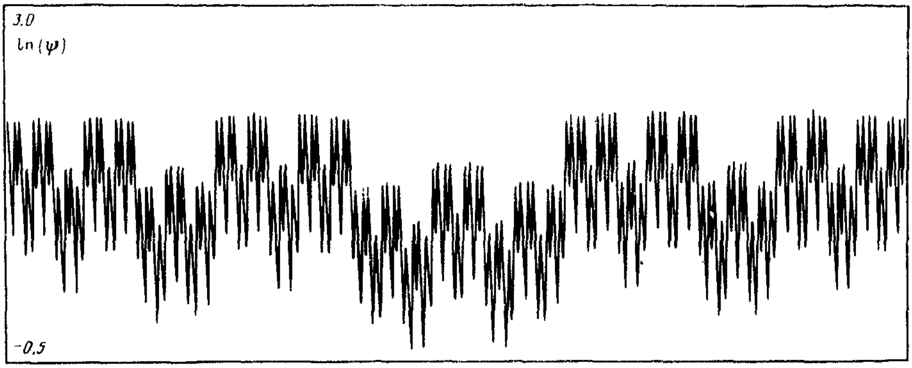

FIG. 2. The ground-state wave function ψ at the nodes 0,...,377 at $J = 10$. The function ψ, which represents a hierarchy of molecules,[2] has oscillations of all scales.

values of the wave function. If the wave function is not to increase too rapidly, we must take the eigenvector of the matrix S which corresponds to an eigenvalue with a modulus less than unity (Fig. 2).

3. We turn now to the calculation of the critical indices. We begin with the field H_{c2}, and we consider a nucleating region of dimension L. The energy density is the sum of the "kinetic" term and a diamagnetic term: $L^{-2\gamma} + H^2L^2$. Minimizing with respect to L, and equating to τ, we find the optimum size of the nucleating region and the field H_{c2}:

$$L \sim H^{-\frac{1}{\gamma+1}}, \qquad H_{c2} \sim \tau^{\alpha}, \quad \alpha = \frac{\gamma+1}{2\gamma} \,. \tag{6}$$

The index α ranges from 1 to 0.5 as J varies from 1 to ∞.

We now seek the temperature dependence of the modulus of the order parameter Δ. Since the wave function of the ground state is highly inhomogeneous ($\delta < 1$), we cannot simply replace $\langle\Delta^4\rangle$ by $\langle\Delta^2\rangle^2$. Instead we should use the relation $\langle\Delta^4\rangle \sim L^{1-\delta}\langle\Delta^2\rangle^2$ (L is the size of the region over which the average is taken), which is valid under the condition $L \lesssim \xi$, where ξ is the correlation length. The ξ dependence of τ is given by $\xi^{-2\gamma} \sim \tau$. Replacing $\langle\Delta^4\rangle$ by $\xi^{1-\delta}\langle\Delta^2\rangle^2$ in (1), we find that $\langle\Delta^2\rangle$

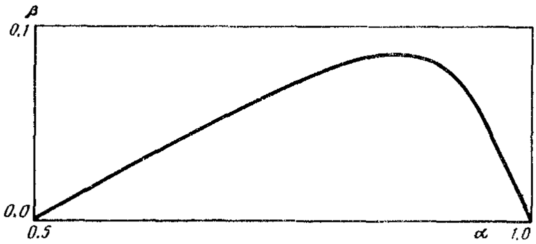

FIG. 3. The index of the heat capacity, β, versus the index of the upper critical field, α.

minimizes the expression $-\tau\langle\Delta^2\rangle + \xi^{1-\delta}\langle\Delta^2\rangle^2$. We can find Δ^2 and the capacity:

$$\langle\Delta^2\rangle \sim \tau^{\beta+1}, \qquad \beta = \frac{1-\delta}{2\gamma}, \qquad c \sim \tau^{\beta}. \tag{7}$$

Knowing the index $\langle\Delta^2\rangle$, we can determine the index of the London depth and find the Ginzburg-Landau parameter κ:

$$\lambda^2 \sim \tau^{-1-\beta}$$

$$\xi_\perp \sim \tau^{-\frac{1}{2\gamma}} \quad \text{— perpendicular to the layers}$$

$$\xi_\parallel \sim \tau^{-1/2} \quad \text{— parallel to the layers} \tag{8}$$

$$\kappa_\perp = \frac{\lambda}{\xi_\perp} \sim \tau^{\mu}, \quad \mu = \frac{1+\delta}{4\gamma} - \frac{1}{2}, \qquad \mu \leqslant 0$$

$$\kappa_\parallel = \frac{\lambda}{\xi_\parallel} \sim \tau^{\nu}, \quad \nu = \frac{\delta-1}{4\gamma}, \qquad \nu \leqslant 0.$$

We find two values for the Ginzburg-Landau parameter: a longitudinal value $\kappa_\parallel$ and a transverse value $\kappa_\perp$. Both increase toward T_c. We see that, regardless of the materials making up the layers, a type II superconductivity occurs near T_c. Consequently, although the critical behavior is not universal, the critical indices depend on only the one parameter J, and the relation between them can be verified experimentally (Fig. 3).

4. It is important to note the distinction between the scaling we are discussing here and that which usually prevails at the point of a second-order transition. In our case, the scaling occurs only as the scale is increased by a factor of Φ; as a result, the functional dependences are more complicated than power laws.[2] For the field H_{c2}, for example, we have

$$H_{c2} = \tau^{\alpha} G(\log_{\Phi}\xi), \qquad \xi = \tau^{-\frac{1}{2\gamma}} \tag{9}$$

The function G is a bounded periodic function of period 1. The scaling laws for the other thermodynamic quantities have the same correction.

We note in conclusion that the Ginzburg-Landau equation does not apply to the situation of Ref. 1, because of the conditions $\xi_0 \gtrsim d_A$, d_B. Our results are therefore only qualitative, and the critical indices must be calculated from a microscopic theory of superconductivity. On the other hand, our approach is valid for describing the experiments of Ref. 3, where again the field H_{c2} was studied, but in that case for a quasiperiodic network of thin superconductors. No deviation from the usual dependence $H_{c2} \sim \tau$ was observed. For the data of Ref. 3 we find $J = 1.118...$ and $\alpha = 0.986...$, i.e., results which indeed are very close to a linear dependence.

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