Do absolutely closed universes exist?

M. A. Markov
Institute of Nuclear Research, Academy of Sciences of the USSR

(Submitted 4 December 1986)
Pis'ma Zh. Eksp. Teor. Fiz. **45**, No. 2, 61–64 (25 January 1987)

Is it true that the universes which are closed in classical mechanics will not be absolutely closed if nature has some limiting length $l_{\min}$? If so, then it would not be possible for a universe to arise "from nothing."

A closed and isotropic universe will expand to a maximum radius (Landau and Lifshitz, 1973)[1]

$$R_{max} = 2\kappa M/3\pi c^2, \tag{1}$$

where κ is the gravitational constant, c is the velocity of light, and M is the "bare" mass of the universe. In one simple case, the mass M would be equal to the sum of the masses of dust particles having a mass η:

$$M = N\eta \ ,$$

where N is the number of dust particles in the given universe. After the maximum radius is reached, the radius of a closed classical universe would decrease without bound.

Let us assume that no spatial length can be smaller than a certain $l_{\min}$. If this circumstance means that the radius of the universe can be determined *only with a precision* $l_{\min}$, then the radius of a universe of mass M would have to be given by[1)]

$$R = R_{max} + l_{min} \tag{2}$$

The classical volume of the universe is given by (Zel'dovich and Novikov, 1971)[2,1]

$$V_0 = 2\pi^2 R_{max}^3 \tag{3}$$

An imprecision in the determination of the radius $R_{\max}$ leads to an imprecision in the determination of the volume[1)]:

$$\Delta V = 2\pi^2[(R_{max} + l_{min})^3 - R_{max}^3] \tag{4}$$

If $l_{\min} \ll R_{\max}$, then

$$\Delta V \sim 6\pi^2 R_{max}^2 l_{min} \tag{5}$$

On the other hand, the average density

$$\rho_0 = \frac{M}{V_0} = \frac{M}{2\pi^2 R_{max}^3} = \frac{M}{2\pi^2\left(\frac{2}{3}\frac{M\kappa}{\pi c^2}\right)^3} \tag{6}$$

would remain essentially unchanged at small values of $l_{\min}$ (if the relation $l_{\min} \ll R_{\max}$ holds). The mass enclosed in this "spherical shell" is given by

$$\Delta m = \Delta V \rho_0 = \frac{9\pi c^2}{2\kappa} l_{min} . \tag{7}$$

If we assume $l_{\min}$ to be on the order of the Planck dimension,[3]

$$l_{min} \sim \sqrt{\hbar\kappa / c^3} \sim 10^{-33} \text{ cm}, \tag{8}$$

then we would have

$$\Delta m \sim \frac{9\pi}{2} \sqrt{\frac{\hbar c}{\kappa}} \tag{9}$$

This result has the generality of a fundamental theorem in the sense that the quantity Δm is a universal value, totally independent of the value of the bare mass of the universe, M. As these estimates show, the fundamental characteristic of a closed universe (the value of M) drops out of the expression derived for Δm.

If the meaning of this Δm is that it is a quantity "of which the universe does not have enough to be absolutely closed," then *any* classically closed universe should appear to an external observer as a particle of mass $\sim\sqrt{\hbar c/\kappa}$ and dimensions $l_{\min} \sim (\hbar\kappa/c^3)^{1/2} \sim 10^{-33}$ cm if there exists an elementary length. This situation regarding a closed universe in the case in which there exists an elementary length can be illustrated in more detail in the following way. We know that a characteristic property of a closed universe is that its total mass is exactly zero at any instant during its expansion or contraction; the total electric charge and the "spin" of a closed universe also turn out to be zero. In the simple case of a dusty universe the gravitational mass defect would be precisely the negative of $M = N\eta$ when all of the dust particles are at rest.

If, however, the imprecision in $R_{\max}$ rules out in principle the possibility that the gravitational mass defects could be comparable to a given bare mass of the particles, then the universe could not be completely closed. The total mass of a part of a classical closed universe at the instant of maximum expansion is given by[2]

$$m_{tot} = {}^4/_3\, \pi \rho_0 R^3_{max} \sin^3\chi \quad , \tag{10}$$

where χ varies from 0 to π. At $\chi = \pi$, the total mass of the universe approaches zero. The surface of "part" of a closed world is given by[2]

$$S = 4\pi R^2_{max} \sin^2\chi \tag{11}$$

The surface also tends toward zero in the limit $\chi \to \pi$. If Δm is a limiting value of the total mass of the universe—a value below which this total mass cannot go—then the quantity

$$\Delta m_{tot} = m_{min} \sim \sqrt{\hbar c/\kappa} = 4\pi R^3_{max} \sin^3 \chi_0$$

would determine the limiting value $\pi - \Delta\chi_{\min} = \chi_0$. This $\Delta\chi$ also determines the deviation from zero of the surface of a classical closed world (11) of arbitrary mass M, i.e., for any classical closed universe.

The entities which we have been discussing here are limiting cases of worlds whose possible existence was first pointed out by Klein (1961).[4] These worlds have come to be known as "semiclosed worlds."[2] It was pointed out a long time ago that if a single electron (or neutrino) were to be "inserted" into a closed universe, then the universe would necessarily be opened up; it would acquire (in the case of the electron) a Reissner-Nordstrom continuation in the exterior. In this case a universe of arbitrary internal dimensions should be observed in the external space as an electrically charged particle with roughly Planckian dimensions and masses. This possiblity may exist for our own universe, if its average density of matter reaches the critical value $p_{\text{crit}} \sim 10^{-29}$ g/cm^3. The spatially semiclosed entities which we are discussing here are analogous in terms of many properties to black holes, and they have in fact been labeled "black holes of the second kind" (Markov, 1974).[5] An electrically charged entity of a black-hole type was studied in 1970 (Markov and Frolov[6]). It was shown that a charged black hole would have to radiate its charge into the external space, reducing the magnitude of this charge to at least $Q \sim 137e$; this situation would imply a corresponding decrease in the mass of the black hole. Subsequently, however, the well-known Hawking theorem, according to which the mass of a black hole could only increase, became a topic of widespread discussion. The universality of the Hawking theorem put us[6] in a very difficult position. A way out of this difficulty was pointed out in a paper (Markov, 1974)[5] at the Warsaw Gravitational Conference, in which it was asserted that Hawking's theorem breaks down in the quantum region. The result of Ref. 6 was interpreted as an effect of the creation of electron-positron pairs in the Coulomb field of a black hole. After this creation, one of the components of the pair goes off to infinity, reducing the mass of the black hole. Soon thereafter (1974), Hawking formulated the nature of the radiation from black holes in the general case.

The Hawking radiation formulas, however, are valid for macroscopic black holes. If black holes have masses close to the Planck mass (e.g., $m = nm_p$, where $n \sim 1$), the nature of the Hawking radiation of a black hole should change substantially; in particular, the radiation spectrum should become discrete. The final state of an ordinary black hole (a black hole of the first kind) remains an open question in the theory of Hawking radiation.

A question which remains unresolved is whether a black hole radiates itself away entirely or—a possibility for which there are also some grounds—the mass of the final state of the black hole does not disappear and is instead equal to the mass of a maximon (Mal'tsev and Markov, 1980).[7] The final state of a black hole of the second kind (a semiclosed world) must be quite different from that of an ordinary black hole. An ordinary black hole may decay completely in the course of the radiation; at any rate, there is nothing which would formally forbid this possibility. A black hole of the second kind [a "gray" hole in the terminology of Zel'dovich and Novikov (1971)[2]], even in the case of total radiation, would have to continue to exist in the form of a closed universe in principle, no matter what macroscopic dimensions are involved.

We see that these arguments lead to the assertion that the final state of a black hole of the second kind would have to be *stable* and would have a mass on the order of the mass of a maximon, with all the possible astrophysical implications of this situation: The presence of maximons might play the role of the "invisible matter" which is believed to "be lacking" in astrophysics. Stable maximons might combine into unstable, long-lived minimaximon formations (maxideuterium, maxitritium, etc.) and maximon stellar objects ("maximon clusters").[8] The decay of minimaximon systems might also set an upper limit on the energy of the cosmic ray-neutrino spectrum, etc. (Markov, 1981).[9] We should stress here that a hole of the second kind could not arise from a simple collapse of stars. The astrophysical difficulties regarding the presence of a relatively large number of maximons (Polnarev and Khlopov, 1985)[10], if they arise here, common to the existence of stable heavy particles—monopoles, etc.—might find a solution in general in an inflationary version of the universe. If we can establish that these stable maximons do not exist, then the arguments above (if correct) would mean that nature has no fundamental length $l_{\min}$. The comments above also apply to lengths $l_{\min} \gg l_p$ (Ginzburg and Frolov, 1976),[11] in that case in the range of applicability of present-day quantum theory.

[1)] The plus sign in Eqs. (2) and (4) is chosen to make the total energy at the universe positive: $m_{\rm tot} \geqslant 0$.

[1] L. D. Landau and E. M. Lifshitz, Teoriya polya, Nauka, Moscow, 1973 (The Classical Theory of Fields, Pergamon, New York, 1976).

[2] Ya. B. Zel'dovich and I. D. Novikov, Stroenie i evolyutsiya Vselennoĭ (Structure and Evolution of the Universe), Moscow, 1975, p. 136.

[3] M. A. Markov, Preprint P-0187, Inst. Nucl. Res. Ac. Sci. USSR, 1980.

[4] W. O'Klein, Heisenberg und die Physik Unserer Zeit, Braunschweig, 1961.

[5] M. A. Markov, in Gravitational Radiations and Gravitational Collapse, ed. C. De Witt, 1974.

[6] M. A. Markov and V. P. Frolov, Teor. Mat. Fiz. **3**, 41 (1970).

[7] V. Mal'tev and M. Markov, Preprint P-160, Inst. Nucl. Res. Ac. Sci. USSR, 1980.

[8] M. A. Markov, "On the upper limit of the cosmic-ray energy spectrum," Preprint P-0197, Institute for Nuclear Research, Academy of Sciences of the USSR, 1981.

[9] M. A. Markov, "Big bang, small bang, mini bang," Preprint P-0207, 1981.

[10] A. G. Polnarev and M. Yu. Khlopov, Usp. Fiz. Nauk **145**, 369 (1985) [Sov. Phys. Usp. **28**, 213 (1985)].

[11] V. L. Ginzburg and V. P. Frolov, Pis'ma Astron. Zh. **2**, 515 (1976) [Sov. Astron. Lett. **2**, 201 (1976)].

Translated by Dave Parsons