Self-effect and maximum contraction of optical femtosecond wave packets in a nonlinear dispersive medium

E. A. Golovchenko, E. M. Dianov, A. N. Pilipetskiĭ, A. M. Prokhorov, and V. N. Serkin
Institute of General Physics, Academy of Sciences of the USSR

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A generalization of the method of slowly varying amplitudes to nonlinear optics of femtosecond pulses makes it possible to determine the limits of applicability of the nonlinear Schrödinger equation, the maximum contraction of the pulses, and the conditions for the decay of a wave packet into "color" solitons.

The method of slowly varying amplitudes is one of the most effective research methods of the wave theory. This method is based on the assumption that the complex amplitude of the wave varies slowly on a scale of the average oscillation period T and average wavelength.

In the linear-quasioptics approximation, the self-interaction of the wave packets with the narrow frequency and angular spectrum is customarily described by a nonlinear parabolic equation for a complex packet envelope (a nonlinear Schrödinger equation).

Nonlinear wave optics currently faces a fundamentally new problem. It involves the development of the theory of propagation of wave packets under extreme conditions, where their envelope contains only a few oscillation periods of the field and the nonlinear response of the vector of the electric induction of the medium can no longer be described in a quasistatic approximation.[1,2]

We will generalize the method of slowly varying amplitudes to nonlinear optics of femtosecond pulses. It was predicted that the self-steepening of pulses is limited by the initial length and intensity of the pulses and the same limitation was applied to the femtosecond "color" solitons.

In a higher-order approximation of the method of slowly varying amplitudes, the nonlinear dynamics of the wave packets can be described by the equation

$$i\frac{\partial \Psi}{\partial z} = -\frac{1}{2}\,\mathrm{sign}(k_2)\frac{\partial^2 \Psi}{\partial \tau^2} + R\,|\Psi|^2\,\Psi$$

$$-\frac{1}{2}\gamma\frac{\partial^2 \Psi}{\partial z\,\partial \tau} + i\beta\gamma\frac{\partial^3 \Psi}{\partial \tau^3} - i\gamma R\frac{\partial}{\partial \tau}(|\Psi|^2\,\Psi), \qquad (1)$$

which is derived from Maxwell's equations and which takes into account small linear dispersion effects of third order in the parameter T/τ_0 and nonlinear dispersion.[1,2]

Here we use the following nonlinear parameters: $\Psi = E/E_0$, $z = z/z_g$, $\tau = (t - z/v)/\tau_0$, $z_g = \tau_0^2/|k^2|$, $z_{fcm} = n_0/k_0 n_2 |E|^2$, $R = z_g/z_{fcm}$, $\gamma = T/\pi\tau_0$, $\beta = \text{sign}(k_2)(1/4) + (1/6)(k_3/k_2)(\pi/T)$, and $T = 2\pi/\omega_0$, where E_0 and τ_0 are the initial amplitude and length of the wave-packet envelope, $k_2 = \partial^2 k/\partial\omega^2$, and $k_3 = \partial^3 k/\partial\omega^3$.

The values of the principal parameters R and γ in model (1), for which the nonlinear Schrödinger equation no longer holds ($\gamma \equiv 0$), are estimated to be

$$\left|\frac{\partial\Psi}{\partial z}\right| \sim R; \left|\frac{\partial^2\Psi}{\partial\tau^2}\right| \sim R; \quad \left|\gamma R\frac{\partial}{\partial\tau}(|\Psi|^2\Psi)\right| \sim \gamma R\sqrt{R}, \tag{2}$$

$$\left|\frac{1}{2}\gamma\frac{\partial^2\Psi}{\partial z\partial\tau}\right| \sim \gamma R\sqrt{R}; \quad \left|\gamma\beta\frac{\partial^3\Psi}{\partial\tau^3}\right| \sim \gamma R\sqrt{R}.$$

Estimates (2) make it possible to identify the role of the individual physical processes described by (1) in the general picture of the self-effect of the wave packet and to determine the condition for the applicability of the nonlinear Schrödinger equation in the problems of nonlinear optics of femtosecond light pulses:

$$\gamma\sqrt{R} \equiv \gamma N \equiv \frac{T}{\pi\tau_0}\sqrt{P/P_{cr}} \ll 1, \tag{3}$$

where $N = \sqrt{P/P_{cr}}$, and P_{cr} is the critical power level at which the "fundamental" soliton forms.[1,2]

Physically, Eq. (3) implies that the nonlinear broadening of the pulse spectrum may be compared with the carrier frequency ω_0.

In a nonlinear Schrödinger equation model, the propagation of the N-soliton wave packet is accompanied by the formation of a structural feature in the region of the "nonlinear focus"—the formation of an intensive, narrow peak against the background of a broad pedestal (Fig. 1).

We wish to emphasize that the nonlinear Schrödinger equation imposes no constraints on the length of the pulse at the maximum self-steepening point: $\tau_{foc} \sim (4N)^{-1} \to 0$. We used model (1) and mathematical modeling to study the physical mechanism for the restriction of the pulse length at the "focus" and the qualitative representation of the transition to the maximum length, $\tau_{max} \sim T$ (transition to the optical videopulse, which does not contain an rf component of the field). A numerical experiment made it possible to determine the critical value of the parameter $\gamma\sqrt{R}$, at which point the approximation based on the nonlinear Schrödinger equation is no longer valid. Cardinal changes of the pulse dynamics at the nonlinear focus occur at the following values of the principal parameters: $0.1 \leqslant \gamma\sqrt{R} \lesssim 0.6$ (Figs. 1 and 2). An increase in $\gamma\sqrt{R}$ restricts the pulse contraction due to dispersion and gives rise to a strong asymmetry in the shape of the spectral and temporal envelopes of the packet: The center of gravity of the packet is shifted toward the trailing edge and the Stokes–anti-Stokes wings grow in the spectrum (Fig. 1).

The plot of τ_0/τ_{foc} at the focus versus the number of solitons N, calculated for

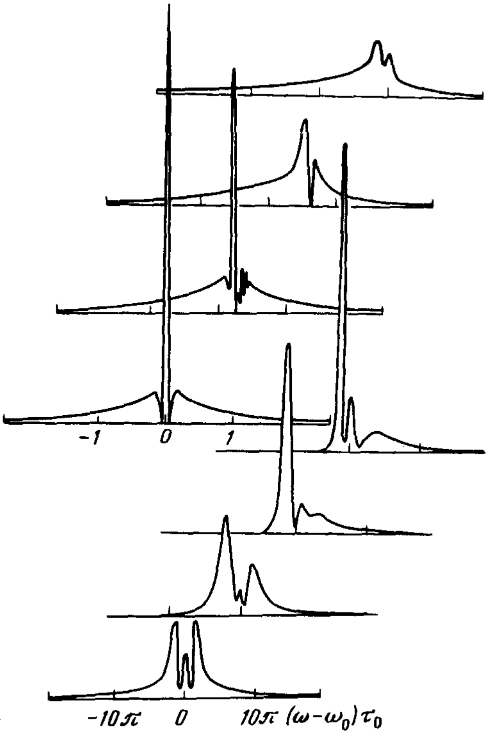

FIG. 1. The envelope and spectrum of a pulse $\Psi(z=0,\tau) = \text{sech}\,\tau$ in the region of "nonlinear focus," calculated for $R = N^2 = 225$, $\beta = 0.25$. a) $\gamma = 0$, $z_{\text{foc}} = 0.0413$; b) $\gamma\sqrt{R} = 0.1$, $z_{\text{foc}} = 0.0407$; c) $\gamma\sqrt{R} = 0.3$, $z_{\text{foc}} = 0.0405$; d) $\gamma\sqrt{R} = 0.6$, $z_{\text{foc}} = 0.0375$.

different values of the parameter $\gamma\sqrt{R}$, reaches the "saturation point" at $\gamma\sqrt{R} \gtrsim 0.3$ (Fig.2a). The minimum length of the pulses steepened at the focus, which was obtained in numerical experiments, is $\tau_{\text{foc}}^{\text{min}} \gtrsim (2\text{–}3)T$.

In the spectral region $k_2 > 0$, the broadening of the pulse spectrum is accompanied by a decrease in the intensity and self-stretching of the wave packet. Analysis of the pulse dynamics in terms of (1) shows that, in contrast with the nonlinear Schrödinger equation model, the spreading of the packet is accompanied by the formation of an envelope at the trailing edge of the shock wave and a strong spectrum asymmetry (Fig. 2b). This effect limits the maximum pulse contraction of screen compressors. A generalization of the method of slowly varying amplitudes to include the nonlinear optics of femtosecond pulses thus allows us to correctly describe the experiments and to suggest methods for achieving the maximum pulse length, $\tau_{\max} \sim 2\text{–}3T$. A compression at the nonlinear focus, $z_{\text{foc}} \simeq 1/2N$, imposes constraints on the maximum power level of the wave packet, $\gamma\sqrt{P/P_{\text{cr}}} \leqslant 0.1$. The maximum pulse length, $\tau_{\text{foc}} \simeq 2\text{–}3T$, can be achieved if the condition $\gamma\sqrt{R} \ll 1$ is satisfied.

Another method of achieving the maximum pulse lengths is based on our analysis of the solution of (1) behind the focus, where the bound states of the solitons of the nonlinear Schrödinger equation, $\Psi = N\,\text{sech}\,\tau$, decay to the solitons of Eq. (1).

A decay of this sort at $\gamma\sqrt{R} \ll 1$ is illustrated in Fig. 3. The duration of the solitons

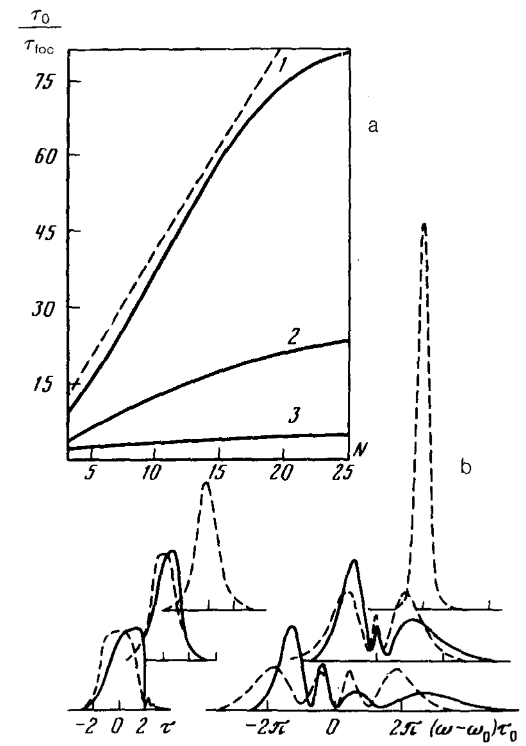

FIG. 2. a) Degree of contraction of the wave packets versus the number of solitons N, calculated for $\gamma = 0$ (dashed line), $\gamma\sqrt{R} = 0.1$ (curve 1), $\gamma\sqrt{R} = 0.3$ (curve 2), $\gamma\sqrt{R} = 0.6$ (curve 3). b) Dynamics of the temporal envelope and of the pulse spectrum $\Psi(z = 0,\tau) = \text{sech}\,\tau$ in the spectral region $k_2 > 0$, calculated for $R = 225$, $\gamma\sqrt{R} = 0.6$, $\gamma = 0$ (dashed lines), $z = 0$, 0.03, 0.06.

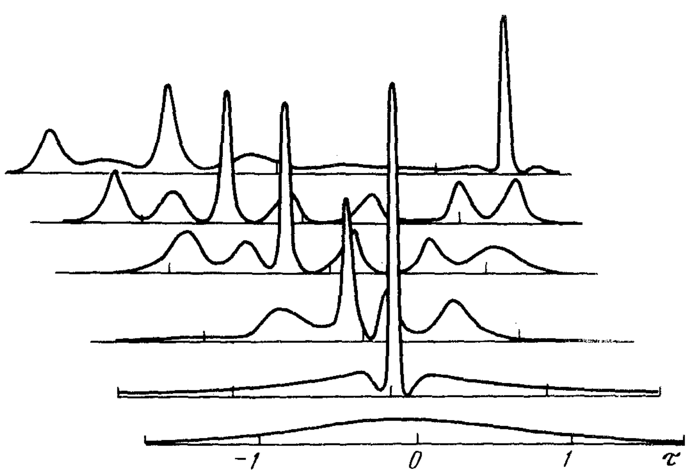

FIG. 3. Decay of a bound state of an $N = 10$ soliton pulse of the nonlinear Schrödinger equation for $R = 100$; $\gamma\sqrt{R} = 0.12$.

formed in (1) is determined by the degree of contraction of the wave packet at the nonlinear focus (Figs. 1 and 3). Singling out the individual pulses in the numerical experiments has revealed that the frequency of these pulses is shifted monotonically toward the "blue" and "red" regions of the spectrum and that the "blue" solitons are formed at the leading edge of the wave packet and the "red" solitons are formed at its trailing edge (Fig. 3).

In the experimental study of the effects we are considering here, it would be useful to take advantage of the unique opportunity offered by the optical fibers: The higher-order dispersion effects can be minimized in optical fibers by means of waveguide dispersion, and the zero-dispersion point ($\partial^2 k/\partial\omega^2|_{\omega_0} = 0$) can be shifted toward longer wavelengths. For example, by placing an optical fiber, in which the zero-dispersion point [$D = (3\text{–}5\ \text{ps/nm}\cdot\text{km})$] is displaced toward the long-wave region of the spectrum, at the output of a soliton laser [$\lambda = \simeq 1.5\ \mu\text{m}$; $\tau_0 \simeq 100$ fs (Ref. 3)] it would be possible to detect "color" solitons even in the first ten meters of the optical fiber.

We wish to stress that the spectrum of pulses with a maximum length, $T_{\max} = 2\text{–}3T$ (10–20 fs, $\Delta\nu = 1000\text{–}500\ \text{cm}^{-1}$) completely overlaps the line for the Raman scattering in glass. This means that an effective stimulated Raman scattering can be achieved. Specifically, self-conversion of a pulse can be achieved when its blue spectral components function as the pump relative to the red components.[1]

Our numerical experiments showed that stimulated Raman self-scattering of pulses in the region of maximum self-steepening of pulses (Fig. 1) leads to the decay of the bound states of solitons of the nonlinear Schrödinger equation and is not an obstacle to the achievement of the maximum pulse length $T_{\max}$.

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