Critical superfluid spin current in ^{3}He-B

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(Submitted 8 December 1986)
Pis'ma Zh. Eksp. Teor. Fiz. **45**, No. 2, 106–108 (25 January 1987)

Low-frequency dynamic equations are derived for a precessing spin in ^{3}He-B. They are used to derive the spin current flowing through a channel as a function of the phase difference between the ends of the channel. The critical values of the current and of the phase gradient, at which there is a transition from a steady-state flow to a time-varying flow, are found.

Borovik-Romanov *et al.*[1] have studied the flow of a spin current through a channel connecting two volumes containing superfluid ^{3}He-B. In each of the two volumes and also in the channel, the spin **S** precessed in a magnetic field $\mathbf{H}_0$. With increasing difference in the precession phases in the two volumes, it was found that there was a transition from a steady-state flow of the spin to a time-varying flow, accompanied by a slipping of the precession phase through the channel. In the present letter we offer a theoretical description of the steady-state flow of a spin current through a long channel. We derive limiting values of the current and the phase gradient, up to which such a flow is possible.

Under the experimental conditions of Ref. 1, the flow of the spin current may be regarded as a small perturbation of a spatially uniform spin precession. The motion of the order parameter in the case of such a precession in ^{3}He-B is determined unambiguously by the motion of the spin,[2] which is conveniently described by specifying the angles α and β (Fig. 1). At $\cos\beta > -1/4$, even if the dipole interaction is taken into account, the precession in ^{3}He-B is degenerate wth respect to both angles α and β. This degeneracy is lifted by the gradient energy F_∇. The part of F_∇ of importance below is that which depends on the spatial derivatives transverse with respect to the direction of $\mathbf{H}_0$: $\alpha_{,k} = \partial\alpha/\partial x_k$ and $u_{,k} = \partial u/\partial x_k$, where $u = \cos\beta$ and $k = 1,2$. After an average is taken over times long in comparison with the precession period, this part of F_∇ becomes[3]

$$F_{\nabla\perp} = \frac{1}{2}(1-u)[(1-u)c_\parallel^2 + (1+u)c_\perp^2]\,\alpha_{,k}\alpha_{,k}$$

$$- c_\parallel^2\frac{1-u}{1+u}\left(\frac{3}{1+4u}\right)^{1/2}\alpha_{,k}u_{,k} + \frac{1}{2}\left[\frac{c_\perp^2}{1-u^2} + \frac{3c_\parallel^2}{(1+u)^2(1+4u)}\right]u_{,k}u_{,k}\ . \quad (1)$$

Here $c_\parallel^2$ and $c_\perp^2$ are the squares of the velocities of the spin waves. The gradient energy F_∇ may be regarded as small if the length scale of the inhomogeneities that arise, l, is large in comparison with the dipole length $l_D \sim 10^{-3}$ cm. If we also assume that the frequencies ω_∇, which characterize the deviation of the motion of the spin from a uniform precession, are small in comparison with the dipole frequency Ω, and if we

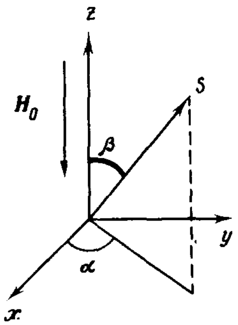

FIG. 1.

expand the Leggett equations in $(l_D/l)^2$ and ω_{∇}/Ω, we find, in the lowest order, a closed system of equations for α and u:

$$\frac{\partial u}{\partial t} + \frac{1}{\omega_L}\frac{\partial}{\partial x_\zeta}\left(-\frac{\partial F_\nabla}{\partial \alpha_{,\zeta}}\right) = 0, \tag{2}$$

$$\frac{\partial \alpha}{\partial t} = -\omega_L + \frac{1}{\omega_L}\left[\frac{\partial F_\nabla}{\partial u} - \frac{\partial}{\partial x_\zeta}\left(\frac{\partial F_\nabla}{\partial u_{,\zeta}}\right)\right] \tag{3}$$

where $\zeta = 1,2,3$; and ω_L is the Larmor frequency. Equation (2) expresses conservation of the longitudinal component of the spin, $S_z = \omega_L u$; the expression acted upon by $\partial/\partial x_\zeta$ is the $z\zeta$ component of the spin superfluid current $j_{z\zeta}$. Equation (3) is analogous to the equation for the phase of the order parameter in a superconductor or in ^{4}He. The angle α is analogous to the phase itself, while ω_L serves as a chemical potential.

During steady-state current flow, we would have $\partial u/\partial t = 0$ and $\partial\alpha/\partial t = -\omega_p$, where ω_p is the precession frequency. Working from the quantities appearing in these equations, we can construct a parameter with the dimensionality of a length, which is unique within the ratio $c_\parallel/c_\perp$: $\xi = c_\perp/\sqrt{\omega_L(\omega_p - \omega_L)}$. This parameter plays here the role which is played by the correlation length in a superconductor. In particular, if we wish to regard the flow as one-dimensional and to ignore edge effects, we should require that the length of the channel be large in comparison with ξ. With $\omega_L = 2\pi\cdot 460$ kHz, $\omega_p - \omega_L = 2\pi\cdot 500$ Hz, and $c_\perp = 15$ m/s, we would have $\xi \sim 10^{-2}$ cm. At $\omega_p > \omega_L$, system (2),(3) has stable, steady-state solutions of the type $\alpha' = \text{const}$, where the prime means the derivative in the direction along the channel, and $u = \text{const}$. Here we would have $u = -1/4$ if $0 \leqslant \alpha' \leqslant \alpha'_{c1}$, where $\alpha'_{c1} = [4c_\perp^2/(5c_\parallel^2 - c_\perp^2)]^{1/2}(1/\xi)$, or $u = [c_\parallel^2/(c_\parallel^2 - c_\perp^2)]\{1 - [c_\perp^2/c_\parallel^2(\alpha'\xi)^2]\}$ if $\alpha'_{c1} \leqslant \alpha' \leqslant \alpha'_{c2}$, where $\alpha'_{c2} = 1/\xi$. These solutions describe the flow of a spin current along a long channel. The magnitude of the current is given by the following expression according to (2):

$$j = -(1-u)[(1-u)c_\parallel^2 + (1+u)c_\perp^2]\,\alpha' \tag{4}$$

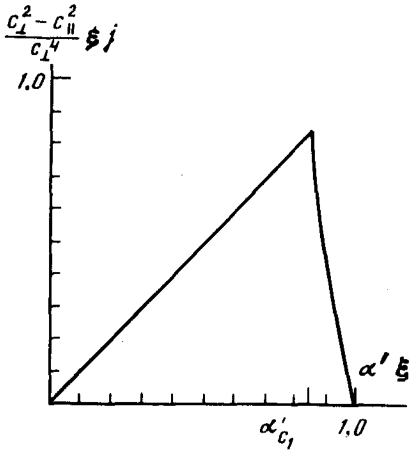

FIG. 2.

Figure 2 shows the current as a function of α'. The current reaches a maximum at $\alpha' = \alpha'_{c1}$. At the point $\alpha' = \alpha'_{c2}$ the angle β and the current vanish. At $\alpha' > \alpha'_{c2}$ there are no steady-state solutions of this type, but at $\beta = 0$ the phase α becomes indefinite, so that one or several "turns of the spiral" may be thrown off, and there is a return to the case $\alpha' < \alpha'_{c2}$.

The quantity α'_{c2} is therefore a critical phase gradient for a given difference between the phases at the ends of the channel. If the experimental conditions instead specify the total current through the channel, then the transition to a time-varying situation will occur at the maximum current, which corresponds to the gradient α'_{c1}. Both of the critical gradients are proportional to $[\omega_L(\omega_p - \omega_L)]^{1/2}$. In the experiments of Ref. 1, the critical difference between the phases at the ends of the channel was observed to increase with increasing $\omega_p - \omega_L$. An estimate of the critical gradient as an average over the length of the channel leads to values which differ from the theoretical values by a factor of about 2. We can thus say that the equations derived here agree qualitatively with the experimental results. A quantitative comparison would be premature at this point because of the inaccuracy of the values of α' found experimentally and also because the theoretical analysis has ignored dissipative processes, which would unavoidably accompany the flow of a superfluid spin current for the method used to observe it.

If a constant difference in precession frequencies, $\delta\omega_p$, is maintained between the ends of the channel, there will be a regime which is periodic in time, in which phase slippage will occur at uniform time intervals satisfying the obvious relation $T = 2\pi n/\delta\omega_p$, where n is the number of turns thrown off in a single event. Such a state is analogous to the resistive state of a superconductor.[4] Note should also be taken of the analogy with the time-varying phenomena which are observed during the flow of a bulk superfluid current in ^{3}He-A (Ref. 5), where the phase slippage is caused by a rotation of the orbital part of the order parameter.[6,7] A quantitative description of the time-varying regime will require explicit consideration of dissipative processes, and it will require going beyond Eqs. (2) and (3).

This study was carried out in close contact with the experiments of Ref. 1. I wish to thank A. S. Borovik-Romanov, Yu. M. Bun'kov, V. V. Dmitriev, and Yu. M. Mukharskiĭ for a stimulating collaboration and A. V. Markelov for an important comment.

[1]A. S. Borovik-Romanov, Yu. M. Bun'kov, V. V. Dmitriev, and Yu. M. Mukharskiĭ, Pis'ma Zh. Eksp. Teor. Fiz., **45**, 98 (1987) [JETP Lett. **45**, 124 (1987)].
[2]I. A. Fomin, Zh. Eksp. Teor. Fiz. **84**, 2109 (1983) [Sov. Phys. JETP **57**, 1227 (1983)].
[3]I. A. Fomin, Zh. Eksp. Teor. Fiz. **78**, 2392 (1980) [Sov. Phys. JETP **51**, 1203 (1980)].
[4]B. I. Ivlev and N. B. Kornin, Usp. Fiz. Nauk **142**, 435 (1984) [Sov. Phys. Usp. **27**, 206 (1984)].
[5]D. N. Paulson, M. Krusius, and J. C. Wheatley, Phys. Rev. Lett. **37**, 599 (1976).
[6]G. E. Volovik, Pis'ma Zh. Eksp. Teor. Fiz. **27**, 605 (1978) [JETP Lett. **27**, 573 (1978)].
[7]J. R. Hook and H. E. Hall, J. Phys. **C12**, 783 (1979).

Translated by Dave Parsons