# Instantons and lattice confinement

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The freezing of the quantum fluctuations of the gauge fields in the lattice is analyzed. The region of the instanton–anti-instanton gas, the region of annihilation of the instantons and the region of the classical solutions are identified. The contribution of single-instanton and many-instanton configurations to the coefficient of tension of the string is 5%.

Confinement, the most interesting property of non-Abelian gauge theories, so far has no theoretical explanation. There are some compelling arguments[1] in support of the fact that confinement is linked with random vacuum fields. This viewpoint is supported by numerical calculations for the lattice.[2] However, the mechanism for the formation of stochastic fields has so far not been explained. Theory has shown that the vacuum in non-Abelian theories has single-instanton and many-instanton configurations which can, in principle, give rise to random fields because of the random distribution of the instantons in the four-dimensional space-time. For the lattice SU(2) gluodynamics we will analyze numerically the contribution of the many-instanton configurations to the linear potential which accounts for the confinement.

Lattice calculations on a computer generate systematic vacuum-field configurations with a probability density of $e^{-S}$ ($S$ is the lattice variant of the Euclidean action of the theory). The topological properties of the vacuum were studied in Refs. 3 and 4 by introducing a technique of "freezing," which made it possible to systematically smooth out the quantum fluctuations. It was shown in Refs. 3 and 4 that the action decreases rapidly after the initial freezing, followed by rather long series of freezing steps during which the action is essentially constant. A remarkable feature of this approach is that $S \approx nS_I$ (where $S_I = 8\pi^2/g^2$ is the action of a single instanton) in this series of steps. Detailed studies show[4] that freezing does indeed single out the $n$-instanton configuration, which subsequently becomes a configuration with a topological charge $n-1, n-2, \ldots$ .

In our calculations we have used a lattice $8^4$ in size for the SU(2) gluodynamics with $\beta = 2.3$ and $\beta = 2.35$ ($\beta = 4/g^2$). With these values of $\beta$ for the $8^4$ lattice, the effects arising from the finite size of the system are tolerable, and the scaling law for the topological susceptibility of the vacuum is roughly satisfied.[3] Analysis of the freezing has made it possible to identify the following steps. The action $S$ decreases 50 to 200 fold after the first few freezing iterations, and regions of positive and negative charge can be identified on the projection of the topological-charge density on the planes $xy$, $xz$, $xt$, etc. The instantons and anti-instantons are annihilated in such an instanton–anti-instanton gas after 5 to 50 iterations. The rate at which the action diminishes is reduced during annihilation. A further freezing gives rise to a series of long-lived vacuums with the actions $nS_I, (n-1)S_I, \ldots$ . The particular features of the

  

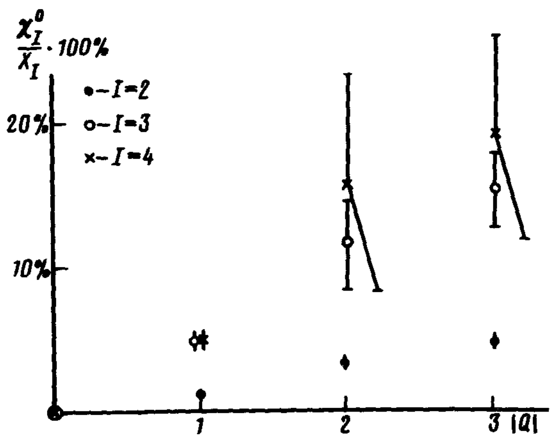


FIG. 1.

cooling described above make it possible to study the characteristics of the instanton–anti-instanton gas, the size distribution of single instantons, and other features. We discuss below the results of the measurement of the Creutz relation,[5] $\chi_I$, where $I$ is the linear dimension of the contour on which the calculation is carried out. The quantity $\chi_I$ is proportional to the coefficient of the linearly increasing term in the quark-antiquark potential. Figure 1 is a plot of $x = \chi_I^0/\chi_I$ as a function of the modulus of the topological charge of the vacuum $-|Q|$ for $\beta = 2.3$. The quantity $\chi_I^0$ is the same as $\chi_I$, but one that is calculated in the "frozen vacuum," i.e., in the first quasistable state that arises during the freezing. Since we could not see a dependence of $\chi_I$ on $Q$ within the error limits of our calculations, we conclude that the functional dependence $x(Q)$ is completely related to $\chi_I^0(Q)$. Figure 1 shows that the coefficient of tension of the string increases with increasing $Q$ and that the tension of the string increases with increasing size of the contour $I$. These observations are consistent with the intuitive picture of the formation of stochastic fields due to the (anti) instantons which are randomly scattered in the four-dimensional lattice. The contribution of instanton configurations to the confinement may appear to be quite substantial—up to 20%— for $I = 4$ and $|Q| = 3$. The larger the $|Q|$, however, the fewer times is such a configuration encountered in the sequence of the vacuum configurations played out on the computer. A qualitative understanding of the situation therefore requires the analysis of the quantity

$$\bar{X}_I = \Big(\sum_{Q=0;\,\pm 1;\ldots} X_I(Q) N_Q\Big) \Big/ \sum_{Q=0;\,\pm 1;\ldots} N_Q$$

Here $N_Q$ is the number of states with a topological charge $Q$. It turns out that $\bar{X}_I = 1\%$, $4.5\%$, and $5\%$ for $I = 2$, 3, and 4. In the case of the value considered by us, $\beta = 2.3$, we know that $\chi_3$ and $\chi_4$ lie in the "scaling corridor," i.e., they describe the coefficient of tension of the string. We repeated our calculations for $\beta = 2.35$. The conclusion remained the same even in this case. A total of 78 configurations was analyzed for $\beta = 2.3$ and 118 configurations were analyzed for $\beta = 2.35$. To render these configurations statistically independent, we divided them by 15 Monte Carlo iterations. As a result of all our calculations, we conclude that the contribution of instantons to the confinement is about 5%.

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