Magnetoelectric waves

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In conducting ferromagnets the nonuniform alternating magnetic field directed along the magnetic moment of the crystal gives rise to an electric field which is directed along the wave vector.

A nonuniform magnetoelectric effect can occur in conducting magnetic materials, regardless of their crystallographic symmetry. This effect can be summarized by saying that a nonuniform electric field causes the appearance of an inhomogeneity in the magnetization of a crystal.[1] We will discuss here the inverse effect, where the external

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nonuniform magnetic field accounts for the nonuniform distribution of the conduction electrons. This effect stems from the fact that at $T \neq 0$ this field causes the magnetization inhomogeneity of the crystal. Since the energy of the electrons depends on the magnetization, they tend to collect in those regions where they have a minimum energy, going there from the regions where they have a maximum energy.

In contrast with Ref. 1, where the direct magnetoelectric effect was studied only in a static field, we analyze the inverse effect in alternating fields. We will show that an external periodic magnetic field causes the appearance of distinctive magnetoelectric waves in the crystal. In contrast with the ordinary electromagnetic waves, the electric vector in them is directed along the wave vector of the wave. The latter is generally oriented in an arbitrary manner relative to the magnetic vector which is parallel to the magnetic moment of the crystal.

The calculations were carried out for a ferromagnetic semiconductor. Since the current is proportional to the gradient of the electrochemical potential of electrons, μ, we first must determine how μ depends on $\mathscr{H}$. In a spin-wave region, the mean field of the magnetized crystal causes a Zeeman splitting of the conduction band into two spin subbands separated by an amount $\approx AS$, where A is the integral of the s-f exchange, and S is the magnitude of the f spin. Under conditions typical of a semiconductor all electrons are situated in the lower subband. In the leading order in $1/S$, the energy of these electrons is given by

$$\mathcal{E}_{\mathbf{k}} = \frac{k^2}{2m} - \frac{AS}{2} + \frac{A}{2N}\sum_{\mathbf{q}} \frac{n_{\mathbf{q}}[(\mathbf{k}+\mathbf{q})^2 - k^2]}{2mAS + (\mathbf{k}+\mathbf{q})^2 - k^2} - \mu_B \mathcal{H}$$
$$n_{\mathbf{q}} = \left\{ \exp\left(\frac{\mu_B \mathcal{H} + \omega_{\mathbf{q}}^0}{T}\right) - 1 \right\}^{-1}, \qquad (1)$$

where k and m are the momentum and the effective electron mass, $\omega_{\mathbf{q}}^0$ is the magnon frequency in the absence of a field, $\mathbf{q}$ is the magnon quasimomentum, μ_B is the Bohr magneton, $\hbar = 1$, $A > 0$, and $S \gg 1$.

Expanding (1) with respect to the field, we find the following expression for the force $\mathscr{F}$ which acts on the electron in a uniform magnetic field.

$$\mathscr{F} = \kappa e \nabla \mathcal{H}, \qquad (2)$$

$$\kappa = \left\{ \frac{3\sqrt{2}\,\zeta\left(\frac{3}{2}\right) a^3 M^{5/2} T^{3/2}}{16\pi^{3/2} Sm} + 1 \right\} \frac{\mu_B}{e} \quad \text{for } T \ll \frac{mAS}{M}, \qquad (3)$$

$$\kappa = \left\{ \frac{\sqrt{A}\, T M^2 a^3}{2\pi\sqrt{2mS}} + 1 \right\} \frac{\mu_B}{e} \quad \text{for } T_c \gg T \gg T_c/S. \qquad (4)$$

Here M is the effective magnon mass ($Ma^2 \sim S/T_c$, where T_c is the Curie temperature), a is the lattice constant, and $\zeta(n)$ is the Riemann zeta-function. If conditions

(3) and (4) are satisfied, we assume that the width of the conduction band, $W \sim 10/ma^2$, is large in comparison with AS.[1]

Near T_c the electron energy can be expressed directly in terms of the magnetization[2] η:

$$\mathscr{E}_{\mathbf{k}} = \frac{k^2}{2m} - g\eta^2 - \mu_B \mathscr{H}, \qquad g \sim (AS)^{2/3} W^{1/3} . \tag{5}$$

According to (5), the quantity κ in this case is given by

$$\kappa = (2g\chi\eta_0 + 1)\frac{\mu_B}{e}, \qquad 0 < \eta < S , \tag{6}$$

where η_0 is the magnetization at $\mathscr{H} = 0$.

In the limit $T \to T_c$, the factor κ diverges, according to (6), as $(T_c - T)^{\beta - \gamma}$; i.e., it diverges as $(T_c - T)^{-1}$ if the critical exponents for the susceptibility and magnetization are $\gamma = 4/3$ and $\beta = 1/3$. Far from T_c, however, the effect is large because the indirect action of the magnetic field on the electron through the magnetization of the crystal is much stronger than its direct action. This behavior is attributable to the fact that the conduction electron is coupled to the magnetic subsystem of the crystal by a very strong s-f exchange, $AS \gg T_c$. [Equation (3) does not contain AS because the electron spin is adjusted to match the local magnetic moment of the crystal. This also applies to the other electronic characteristics in this temperature range.[1]] For $AS \sim 1$ eV, $W \sim 10$ eV, and $T_c \sim 10^{-3}$ eV, for example, the value of κ in (4) is four orders of magnitude higher than μ_B/e at $T \sim T_c/S$. For $\nabla H \sim 10^4$ Oe/cm the action of the magnetoelectric force (2) on the electron is the same as that of an electric field of 1 V/cm.

Let us now consider the periodic processes in a magnet which are caused by a periodic nonuniform magnetic field. We assume that the frequency of the field, ω, is sufficiently low, so that, first, the eddy currents and fields can be ignored. Accordingly, the field is assumed to penetrate the conductor deeply compared with its dimensions. Secondly, the local magnetization at such ω is assumed to follow adiabatically the value of $\mathscr{H}$ at this point. We can thus use (2), and we can write the equations for the current $\mathbf{j}$ and the electric field $\vec{\mathscr{E}}$ in the form

$$\begin{aligned} &\mathbf{j} = \sigma\left(\mathscr{E} + \frac{\kappa}{e}\nabla\mathscr{H} - \lambda\nabla\rho\right), \qquad \lambda = \frac{1}{e^2}\frac{d\mu}{dn} \\ &\partial\rho/\partial t = -\operatorname{div}\mathbf{j}, \quad \operatorname{div}\vec{\mathscr{E}} = 4\pi\rho/\epsilon , \end{aligned} \tag{7}$$

where σ is the conductivity, ϵ is the dielectric constant, ρ is the charge density, and n is the average electron density.

If the magnetic field is spatially periodic, the current and the electric field will be directed, as follows from (7), along the wave vector $\mathbf{k}$, regardless of the direction of the magnetic field, if the eddy currents are ignored. We obtained from (7) the following values for the amplitudes of the electronic field and current:

$$e\mathcal{E} = ik\kappa\mathcal{H}\left(1 + \frac{\epsilon k^2\lambda}{4\pi} + \frac{i\omega}{\omega_M}\right)^{-1}, \tag{8}$$

$$j = -\frac{i\epsilon\omega\mathcal{E}}{4\pi}, \quad \omega_M = \frac{4\pi\sigma}{\epsilon}. \tag{9}$$

The magnetoelectric effect can also be seen under conditions where the magnetic field in the medium becomes nonuniform due to the skin effect. If, for example, the magnetization of the sample and the external alternating magnetic field $\mathscr{H}_y$, which is parallel to the magnetization, lie in the (x,y) plane bounding the sample, damping of the field $\mathscr{H}_y$ deeply into the bulk of the sample will give rise to an electric field $\mathscr{E}_z$ which is perpendicular to the surface of the sample. Determining the field $\mathscr{H}_y(z)$ from the standard equations of the skin-effect theory and using (7) and the boundary condition

$$j_z|_0 = \frac{\epsilon}{4\pi}\frac{\partial\mathcal{E}_z}{\partial t}|_0,$$

which follows from the equations of continuity and Poisson's equation, we obtain the following equation for the magnetoelectric strength:

$$e\mathcal{E}_z = \frac{\omega_M\alpha\kappa\mathcal{H}_y(0)e^{-\alpha z}}{\alpha^2\sigma\lambda + i\omega + \omega_M}, \quad \alpha = \left[\frac{4\pi\mu\sigma\omega i}{c^2}\right]^{1/2}. \tag{10}$$

The current j_z is, as before, given by (9). According to (10), the field $\mathscr{E}_z$ is of order $4\pi\sigma\kappa/c$ of the ordinary vortex field $\mathscr{E}_x$; i.e., as $T\to T_c$, the former is, according to (6), clearly larger than the latter. In this limit, the skin depth α^{-1} is small even at low values of ω.

[1)]The condition under which Eq. (4) can be used is the usual quasiclassical requirement that the field change only slightly at the electron wavelength. The use of Eq. (3) additionally requires that the field vary slightly also at the thermal wavelength of the magnon.

[1]E. L. Nagaev, Physics of magnetic semiconductors, Mir, Moscow, 1983, p. 386.
[2]E. L. Nagaev, Zh. Eskp. Teor. Fiz. **90**, 652 (1986) [Sov. Phys JETP **63**, 379 (1986)].

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