Spectrum of relaxation times in disordered conductors

B. L. Al'tshuler, V. E. Kravtsov, and I. V. Lerner

B. P. Konstantinov Institute of Nuclear Physics, Academy of Sciences of the USSR; Institute of Spectroscopy, Academy of Sciences of the USSR

(Submitted 30 December 1986)

Pis'ma Zh. Eksp. Teor. Fiz. **45**, No. 3, 160–163 (10 February 1987)

The response of a disordered conductor to an external-field pulse with a δ-function temporal profile decays in accordance with $\exp[-(\ln^2 t)/8u]$ at long times t. This result means that the conductor has a spectrum of relaxation times, whose distribution has a logarithmically normal asymptotic behavior.

1. The conductivity σ of a disordered metal is described as a function of the frequency ω in the classical case by the Drude formula:

$$\sigma(\omega) = \sigma_0(1 - i\omega\tau)^{-1}. \tag{1}$$

The quantum correction to $\sigma(\omega)$ is[1,2]

$$\delta\sigma = -(2e^2D/\pi\hbar)\frac{1}{L^d}\sum_{\mathbf{q}}(D\mathbf{q}^2 - i\omega + 1/\tau_\varphi^0)^{-1}. \tag{2}$$

Here $D = l^2/\tau d$ is the diffusion coefficient in the space of dimensionality $d = 2 + \epsilon$, l and τ are respectively the mean free path and the mean free time, and $\tau_\varphi^0 \propto T^{-p}$ is the time scale of the phase relaxation due to inelastic processes[2] ($\tau_\varphi^0 \gg \tau$).

When we take both the classical and quantum contributions, (1) and (2), respectivlely, into account, we find the response to an external-field pulse with a δ-function temporal profile, to be

$$\sigma(t) = \int \sigma(\omega)\, e^{-i\omega t} d\omega/2\pi, \tag{3}$$

$$\sigma(t) = \frac{\sigma_0}{\tau} e^{-t/\tau} - \frac{e^2}{2\pi^2\hbar} t^{-d/2}(4\pi D)^{-\epsilon/2} e^{-t/t_\varphi^0}. \tag{4}$$

Here we have $t_\varphi^0 = \min(\tau_\varphi^0, L^2/D)$ for a sample of size L with massive contacts or $t_\varphi^0 = \tau_\varphi^0$ for an isolated sample. In the interval $\tau < t < t_\varphi^0$ the classical contribution in (4) is small, and the response function undergoes a power-law decay. At $t > t_\varphi^0$, according to (4), $\sigma(t)$ falls off exponentially. Incorporating the higher-order quantum corrections to $\sigma(t)$ in the standard scaling model of localization[3] can lead to only a renormalization of D in (4). It does not alter the exponential nature of the decay of $\sigma(t)$.

2. In this letter we show that the correct expression for $\sigma(t)$ at large values of t, averaged over realizations of the random potential, can be found only by going beyond the scope of the standard scaling. This correct expression is

$$\sigma(t) \propto -\sqrt{\tau/t}\exp\left[-\frac{1}{8u}\ln^2(t/\tau)\right]; \tag{5}$$

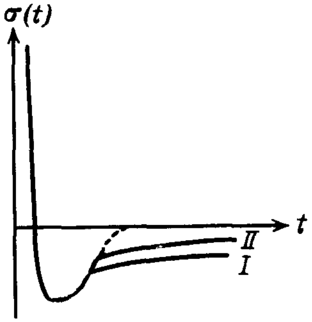

FIG. 1. Response of a conductor to a δ-function external-field pulse for various values of $u(u_{\rm I} > u_{\rm II}$; the dashed line is an exponential decay).

i.e., it falls off with t more rapidly than any power of t^{-1} but considerably more slowly than any exponential function $\exp(-t/t_0)$. The complete response function, described by the sum of expressions (4) and (5), is shown in Fig. 1.

For any dimensionality, the quantity u in (5) is

$$u = \ln(\sigma_0/\sigma), \tag{6}$$

where $\sigma_0 \propto e^2 g_0 l^{-\epsilon}/\hbar$ is the nucleating (classical) value, and $\sigma \propto e^2 g(L) L^{-\epsilon}/\hbar$ the renormalized (observable) value, of the *static* conductivity, and $g_0 \equiv g(l)$. The dimensionless conductance $g(L)$ satisfies the ordinary[3–5] renormalization-group equation with a Gell-Mann–Low function $\beta(g) = \epsilon g - 1$. An important point is that u may be large even in the region in which that expression is applicable. With $d = 2$ and $g \sim 1$, according to (6), we have $u \sim \ln g_0 \gg 1$. At $d > 2$ and $g \gg g_c \approx \epsilon^{-1}$, we have $u = \ln[g_0/(g_0 - g_c)] \gg 1$ in the limit $g_0 \to g_c$. At large values of u, the relaxation of the current is very slow, and, as in the theory of spin glasses, problems may arise due to the fundamental nonequilibrium nature of the system.

3. Corresponding to expression (5) for $\sigma(t)$ is the frequency dependence

$$\sigma(\omega) = \sum_{n=0}^{\infty} C_n (i\omega\tau)^n , \tag{7}$$

where

$$C_n \propto \exp[2u(n^2 + n - 1/2)], \quad n \gg 1. \tag{8}$$

Without the quantum corrections we would have $C_n = 1$, and expression (7) would become (1). A summation of the quantum corrections leads to (8). A similar growth law was observed in Ref. 6 in an analysis of various contributions to $D(\omega)$, and it was proved rigorously in Ref. 7 in a study of the higher-order moments of mesoscopic fluctuations[8,9] of the conductance g and the state density ν. A function of the type in (8) was found previously in a study[10] of the moments of a local state density.

We found growth law (8) as the result of a renormalization-group analysis of a nonlinear σ model which differs from the ordinary model[4,5] in two ways. First, the functional incorporates vertices which are proportional to powers of the small parameter $\omega\tau$. As was shown in Ref. 6, the incorporation of these vertices leads to substantial deviations from single-parameters scaling.[3] Second, the functional incorporates vertices that contain a matrix source; by differentiating with respect to this source, one can directly calculate the conductivity,[11,7] not the diffusion coefficient, as in Refs. 4–6. The analysis was carried out at the temperature $T=0$, at which we have $t_\varphi^0 = L^2/D$. The calculations which lead to (7) and (8) will be published separately.

4. If the relaxation could be characterized by a definite phase relaxation time t_φ^0, then the asymptotic behavior of $\sigma(t)$ in (3) could be only exponential. A nonexponential decay in (5) may be thought of as a manifestation of a wide spread of relaxation times:

$$\sigma(t) \propto - \int_0^\infty \exp(-t/t_\varphi) f(t_\varphi) dt_\varphi \ , \tag{9}$$

where the distribution of relaxation times, $f(t_\varphi)$, has a maximum at $t_\varphi = t_\varphi^0$, according to (4) and (5), and falls off at large t_φ in accordance with the logarithmically normal law $\exp[-\ln^2(t_\varphi/\tau)/8u]$. This behavior of $f(t_\varphi)$ agrees qualitatively with the shape[7] of the distributions of the mesoscopic fluctuations of g and ν.

We have studied the case $T=0$, in which t_φ is the time of the motion of the electron through the sample. For typical paths, this time is indeed on the order of L^2/D. In a disordered system, however, there are of course paths which, if followed, will require considerably more time. Our calculations show that when quantum-mechanical effects are taken into account the relative number of such paths increases sharply, so that they might be called "interference traps." It follows directly from (5) that the distribution of such traps in time is logarithmically normal. With increasing u, this distribution becomes progressively broader.

5. We can assert that the asymptotic result in (5) and the corresponding distribution $f(t_\varphi)$ remain valid at $T \neq 0$, where t_φ is determined by inelastic collisions ($t_\varphi \sim \tau_\varphi$, $L^2 \gg D\tau_\varphi$). The nature of the asymptotic behavior of the distributions of the mesoscopic fluctuations of g and ν, which was found in Ref. 7, was based on certain general properties of the substitution group is apparently universal; i.e., it also holds for fluctuations in any other physical quantities. In particular, the distribution of τ_φ has a logarithmically normal asymptotic behavior. This assertion can be verified by direct calculations incorporating the electron-electron or electron-phonon relaxation mechanism. So far, we have managed to carry out a rigorous calculation only at $\tau_\varphi \gtrsim L^2/D$ (only in this case can we restrict the analysis to the lowest order of a perturbation theory in the inelastic interaction). It is physically clear, however, that in the limit $L \to \infty$ we should examine fluctuations of τ_φ^{-1} not throughout the sample but only in regions of size $L_\varphi^0 \equiv (D\tau_\varphi^0)^{1/2}$. The distribution of fluctuations in the local value of τ_φ, on the other hand, can be found by using the results obtained at $L \lesssim L_\varphi^0$. This approach leads to the asymptotic expression in (5) for the mean $\sigma(t)$.

A distribution analogous to the distribution of τ_φ should also hold for other relaxation times, e.g., the recombination time of electrons and holes in doped semiconductors. We hope that the approach developed here will make it possible to find a *universal* explanation of the results of many experiments on long-time relaxation tails.

We also think that the mechanism which has been found to lead to a distribution of relaxation times may be pertinent to the problem of the $1/f$ noise.

We wish to thank A. G. Aronov, P. B. Vigman, K. B. Efetov, A. I. Larkin, S. P. Obukhov, A. M. M. Pruisken, D. E. Khmel'nitskiĭ, and V. I. Yudson for useful discussions.

[1]L. P. Gor'kov, A. I. Larkin, and D. E. Khmel'nitskiĭ, Pis'ma Zh. Eksp. Teor. Fiz. **30**, 248 (1979) [JETP Lett. **30**, 228 (1979)].

[2]P. W. Anderson, E. Abrahams, and T. V. Ramakrishnan, Phys. Rev. Lett. **43**, 718 (1979).

[3]E. Abrahams, P. W. Anderson, D. C. Licciardello, and T. V. Ramakrishnan, Phys. Rev. Lett. **42**, 673 (1979).

[4]K. B. Efetov, A. I. Larkin, and D. E. Khmel'nitskiĭ, Zh. Eksp. Teor. Fiz. **79**, 1120 (1980) [Sov. Phys. JETP **52**, 568 (1980)].

[5]L. Schäfer and F. Wegner, Z. Phys. B **38**, 113 (1980).

[6]V. E. Kravtsov and I. V. Lepner, Zh. Eksp. Teor. Fiz. **88**, 1281 (1985) [Sov. Phys. JETP **61**, 758 (1985)].

[7]B. L. Al'tshuler, V. E. Kravtsov, and I. V. Lerner, Pis'ma Zh. Eksp. Teor. Fiz. **43**, 342 (1986) [JETP Lett. **43**, 441 (1986)]; Zh. Eksp. Teor. Fiz. **91**, 2276 (1986) [Sov. Phys. JETP 64, No. 6 (1986)].

[8]B. L. Al'tshuler, Pis'ma Zh. Eksp. Teor. Fiz. **41**, 530 (1985) [JETP Lett. **41**, 648 (1985)].

[9]P. A. Lee and A. D. Stone, Phys. Rev. Lett. **55**, 1622 (1985).

[10]F. Wegner, Z. Phys. **36**, 209 (1980).

[11]V. E. Kravtsov and I. V. Lerner, Fiz. Tverd. Tela (Leningrad) **29**, 456 (1987) [Sov. Phys. Solid State (to be published)].

Translated by Dave Parsons