Exotic skyrmions

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Classically stable states with a baryon number equal to the mapping index $n > 1$ have been detected in Skyrme's SU(2) model. The mass distributions of these states have the characteristic shape of a diffuse torus. The moments of inertia are large in comparison with that of the state with $B = 1$.

Skyrme's model of baryons as solitons of a chiral field[1] opened up an approach to the description of the properties and interactions of baryons in which an important role is played by the topological properties of the field configurations.[2–4] It has been established that for the baryon number $B = 1$ an absolute energy minimum is reached in the case of "hedgehog" configurations, for which the SU(2) matrix U, a dynamic variable of the model, is parametrized by a single function of a single variable: $F(R)$, where R is the distance from the center of the soliton.[5] For $B > 1$ the energy of configurations of this type is extremely large, increasing with B as $B(B + 0.87)$ (Ref. 6).

As has been mentioned previously,[7,9] there can be configurations of a different type, in particular, some in which, upon a change in the azimuthal angle φ, the chiral field rotates an integer number of times more rapidly than the radius vector **R** around the symmetry axis (the z axis). This situation corresponds to the parametrization $U = \cos F + i \sin F\vec{\tau}\vec{\pi}$, $\pi_z = \cos\alpha$, $\pi_x = \sin\alpha \cos n\varphi$, $\pi_y = \sin\alpha \sin n\varphi$, where n is an integer (the mapping index), F and α are a profile function and an angular function of the variables z,r, and $z^2 + r^2 = R^2$. The special case of a rapidly rotating skyrmion was analyzed by Weigel *et al.*[7] with $n = 2$; they set $\alpha = \tan^{-1}(r/z)$, as in the spherically symmetric case. The mass of a soliton with $B = 2$ for this ansatz is 2.14 of the mass of a soliton with $B = 1$, M_1.

Our calculations show that in the more general case in which the configurations are described by two functions $F(z,r)$ and $\alpha(z,r)$ the mass of the solitons with the small values of n is smaller than nM_1; i.e., such entities are classically stable. A corresponding property holds in other models, e.g., in the stabilization of the soliton by a term of sixth degree in an arbitrary chiral field (by the square of the baryon number density).

In Skyrme's model the mass of static configurations with this choice of U is

$$M = \pi \int r dr dz \left\{ \frac{F_\pi^2}{4}\left[(F,F) + s_F^2\left((\alpha,\alpha) + \frac{n^2}{r^2}s_\alpha^2\right)\right] + \frac{s_F^2}{e^2}\left[[F,\alpha]^2 + \frac{n^2 s_\alpha^2}{r^2}\left((F,F) + s_F^2(\alpha,\alpha)\right)\right] + \frac{m_\pi^2 F_\pi^2}{2}(1 - c_F)\right\}, \tag{1}$$

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$$(F, F) = F'^2 + \dot{F}^2, \ [F, \alpha] = F'\dot{\alpha} - \dot{F}\alpha', \ F' = \partial F/\partial z, \ \dot{F} = \partial F/\partial r, \text{ etc.}$$

$$s_F = \sin F, \quad c_F = \cos F, \quad s_\alpha = \sin\alpha .$$

With $n = 1$, expression (1) becomes the expression which we used in Ref. 4 to study the interaction of skyrmions in an axisymmetric configuration. The baryon number of the system in this notation is

$$B = \frac{n}{\pi}\int s_F^2 s_\alpha [\alpha, F]\, dr dz. \tag{2}$$

For the configurations in which we are interested here, this number can be calculated; it turns out to be $B = (n/\pi)F(0)$. Configurations in which we have $F(0) = \pi$, $F(\infty) = 0$, and $U = -1$ at only a single point, $\mathbf{R} = 0$, have a baryon number $B = n$. A minimization of the energy by the method used in Refs. 8 and 4 leads to soliton masses of 1660, 2530, 3452, and 4420 MeV with $B = 2$, 3, 4, and 5, respectively, and it leads to $M_n - nM_1 = -70 - 65 - 8.95$ MeV ($F_\pi = 018$ MeV, $e = 4.84$ of the mass of a soliton with $B = 1$, and $M_1 = 865$ MeV; Ref. 3). Since the terms in the expression for the mass density in (1), which are proportional to n^2, contain a factor of $1/r^2$, the configurations with values r larger than in the spherically symmetric case ($n = 1$) are favored from the energy standpoint. As a result, the mass distribution has the shape of a diffuse torus, which thickens and increases in size with increasing $n = B$ (Fig. 1).

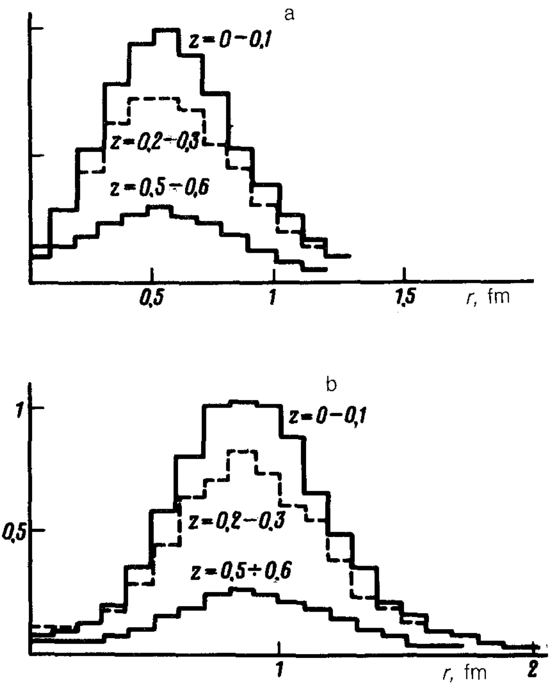

FIG. 1. Mass distribution density as a function of the distance from the symmetry axis for the intervals $z = 0$–0.1, 0.2–0.3, and 0.5–0.6 fm. a—$n = 2$; b—$n = 3$ (arbitrary units).

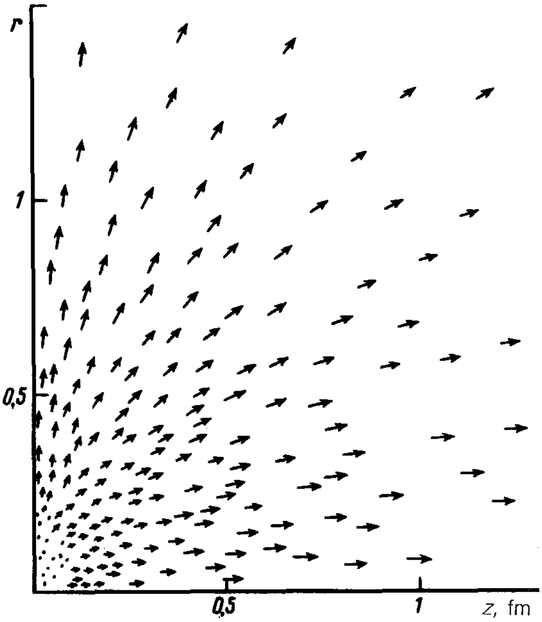

FIG. 2. Chiral field pattern in the (z,r) plane for $n = 2$. The length of the arrows is proportional to $\sin F$.

Figure 2 shows the field distribution in the (z,r) plane; Fig. 3 shows the behavior of the profile function F along the z and r axes.

To calculate the energy of the system as a function of its quantum numbers (the spin and isospin), we need to quantize the rotation, giving up the assumption of spherical symmetry, which was used in Ref. 3. In our case, an isospin rotation by means of the matrices $A(t)$ and a rotation of the coordinate system $R'_i = O_{ik} R_k$, $O_{ik} = \frac{1}{2} \mathrm{Tr}\, A'^{+} \tau_i A' \tau_k$, are not equivalent to each other. The rotation energy is written as follows as a function of the angular velocities:

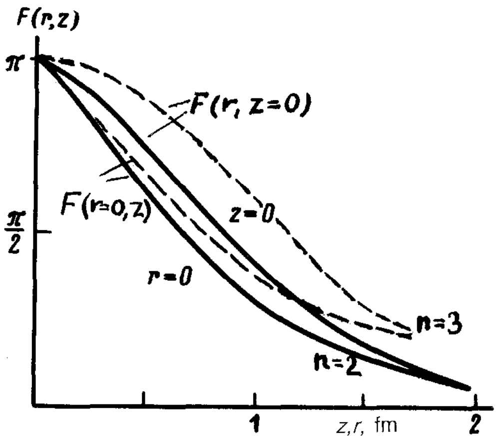

FIG. 3. Change in the profile function $F(z,r)$ along the z axis $(r = 0)$ and along the r axis $(z = 0)$ for $n = 2$ and 3.

$$E_{\rm rot} = \lambda_z(\omega_z + n\Omega_z)^2 + \lambda_r(\omega_x^2+\omega_y^2) + \Lambda_r(\Omega_x^2+\Omega_y^2)$$
$$A^+\dot{A} = i\vec{\tau}\vec{\omega}; \quad A'^+\dot{A}' = i\vec{\tau}\vec{\Omega}, \quad \dot{A} = \partial A/\partial t, \tag{3}$$

where the moments of inertia are[1)]

$$\lambda_z = \frac{1}{2}\int s_F^2 s_\alpha^2 \left\{ F_\pi^2 + \frac{4}{e^2}\left[(F,F) + s_F^2(\alpha,\alpha)\right]\right\} d^3r, \tag{4a}$$

$$\lambda_r = \frac{1}{4}\int s_F^2 \left\{ F_\pi^2(1+c_\alpha^2) + \frac{4}{e^2}\left[(F,F)(1+c_\alpha^2) + (\alpha,\alpha)s_F^2c_\alpha^2 + \frac{n^2}{r^2}s_F^2s_\alpha^2\right]\right\} d^3r, \tag{4b}$$

$$\Lambda_r = \frac{1}{4}\int \Bigg\{ F_\pi^2\left[s_F^2(z\dot{\alpha} - r\alpha')^2 + (z\dot{F} - rF')^2 + \frac{n^2z^2}{r^2}s_F^2s_\alpha^2\right]$$
$$+ \frac{4s_F^2}{e^2}\Bigg[(F,F)(z\dot{\alpha} - r\alpha')^2 + \frac{n^2s_\alpha^2}{r^2}\Big[z^2((F,F) + s_F^2(\alpha,\alpha)) + s_F^2(z\dot{\alpha} - r\alpha')^2$$
$$+ (z\dot{F} - rF')^2\Big] + (z\dot{F} - rF')[(\alpha,\alpha)(z\dot{F} - rF') - 2(z\dot{\alpha} - r\alpha')(\dot{F}\dot{\alpha} + F'\alpha')]\Bigg]\Bigg\} d^3r, \tag{4c}$$

for $n \neq 1$. For $n = 1$, energy (3) also has terms proportional to $\omega_x\Omega_x + \omega_y\Omega_y$. A detailed derivation of these relations will be published separately. For the parameter values specified, we have $\lambda_z = 0.015$ and 0.018 MeV^{-1}, $\lambda_r = 0.021$ and 0.034, and $\Lambda_r = 0.033$ and 0.077 MeV^{-1} for $B = 2$ and 3, respectively. Rotational energy (3) can be expressed in terms of the total angular momentum J and the isospin J_1 of the system as follows ($n = 2$):

$$E_{\rm rot}^{n=2} \simeq [17J_z^2 + 48(J_1^2 - J_{1z}^2) + 30(J_2^2 - J_{2z}^2)] \text{ (MeV)},$$
$$J_{1z} = n\lambda_z\omega_z, \quad J_{2z} = n^2\lambda_z\Omega_z, \tag{5}$$

where $J_2 = J - J_1$. For small values of J and J_1, the rotational energy is below 220 MeV, which is the value above which a state with $B = 2$ becomes unstable. There are corresponding results for $n = 3$ and 4. Consequently, the quantum corrections for the rotation do not change the conclusion regarding the stability of the states found.

Classically stable states also arise if a soliton is stabilized not by a Skyrme term but by the square of the baryon number density B_0^2 or by a linear combination of the Skyrme term and B_0^2. In the case of stabilization by means of B_0^2 for $n = 2$, we have a difference $2M_1 - M_2 \simeq 150$ MeV. The mass distribution becomes more diffuse than in the Skyrme model.

The next step in the study of the states which have been found may be to go to more realistic models, to incorporate vibrational, breathing, and other quantum corrections to the energy, and to study a possible relationship between the configurations found and the configurations corresponding to two skyrmions separated by a particular distance.

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[1)]Expressions for λ_z and λ_ω are given in Ref. 9.

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