# Influence of the quantum effects on the dynamic resistance of tunnel junctions

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The Josephson tunnel junctions which are shunted by a normal resistor and which have normal-phase regions near the barrier are examined. The action which describes these junctions contains two types of terms which characterize the viscosity: quadratic (Gaussian) terms and terms which are strongly nonlinear in the phase operators. The contribution to the dynamic resistance of the junction, which is due to the nonlinear (non-Gaussian) terms in the action, is calculated.

Beginning with the study carried out by Schmid,[1] the dynamic resistance of tunnel junctions (mobility) was studied by several authors.[2–4] Recent experimental studies of the onset of superconductivity in thin granular tin films[5] have revealed that the temperature dependence of the resistance is recoverable. The present theories[6–8] attribute this phenomenon to the quantum-mechanical properties of weak Josephson junctions (with allowance for the viscosity). These junctions are responsible for the transition of the entire sample to the superconducting state. Caldeira and Leggett's[8] model, which corresponds to the shunting of a Josephson junction with a normal-state

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resistor, was analyzed by Fisher.[6] It is conceivable, however, that in the structures such as those studied in Ref. 5 we have a situation where, in addition to a shunt, there is a normal zone near the junction.

In the case of such junctions, the effective action $S$, which describes the dynamics of the transition, along with the term which is quadratic in the phase-difference operators (the Gaussian contribution), contains more strongly nonlinear terms.[9,10] We will calculate the dynamic resistance $R$ of a Josephson tunnel junction, taking into account these nonlinear terms in $S$. The value of $R$ is defined as $(d\overline{V}/dI|_{I\to 0})$: (the bar denotes averaging over time).

In contrast with the calculations by Fisher and Zwerger[3] and Fisher,[6] our calculations are based on the perturbation theory and the use of the action functional over the Keldysh contour[9,10]:

$$S = S_1 + S_H + S_s + S_J; \qquad S_s = S_s^0 + S_s',$$

$$S_1 = \int_{-\infty}^{\infty} dt \left[ \frac{C\dot{\phi}(t)\dot{\chi}(t)}{(2e)^2} + \frac{I(t)\chi(t)}{2e} + \xi(t)\dot{\phi}(t) \right], \tag{1}$$

$$(\dot{x} \equiv d\chi/dt).$$

$$S_H = 4i\int_{-\infty}^{\infty} dt \int_{-\infty}^{\infty} dt' \alpha_0(t-t') \sin\frac{\chi(t)}{4} \sin\frac{\chi(t')}{4} \cos\frac{(\phi(t)-\phi(t'))}{2}$$

$$- 2\eta_0 \int_{-\infty}^{\infty} dt\, \dot{\phi}(t) \sin\frac{\chi(t)}{2}; \qquad S_s' = \eta_s \int_{-\infty}^{\infty} dt\, \dot{\phi}\chi(t); \tag{2}$$

$$S_s = \frac{1}{4}\int_{-\infty}^{\infty} dt \int_{-\infty}^{\infty} dt'\, \alpha_s(t-t')\chi(t)\chi(t'); \qquad S_J = -\frac{I_c}{e}\int_{-\infty}^{\infty} dt \sin\phi \sin\frac{\chi(t)}{2}, \tag{3}$$

where the Fourier transform $\alpha_{0,s}(t)$ is $\tilde{\alpha}_{0,s} = 2\eta_{0,s}\ \omega \coth(\omega/2T)$, $\eta_{0,s} = (4e^2R_{0,s})^{-1}$, $R_0$ and $R_s$ are the resistances of the junction and the normal shunt, $C$ and $I_c$ are the capacitance and the critical Josephson current through the junction, $I(t)$ is the current through the junction, $\xi(t)$ is the source function by means of which the resistance can be determined, and $\phi$ and $\chi$ are respectively the half-sum and the difference of the phase operator on the different branches of the temporal Keldysh contour.[10]

The generating functional must be determined before the correlation functions can be calculated:

$$Z(\xi I) = -\ln W; \qquad W = \int D\phi(t) D\chi(t) \exp(iS).$$

The dynamical resistance in this case is (the bar denotes averaging over time)

$$R = \frac{i}{2e}\frac{d}{dI}\left(\frac{\overline{\delta Z}}{\delta\xi(t)}\right). \tag{4}$$

The renormalization-group equations, which describe the Josephson junction,[1-3] imply a fundamentally different temperature-dependent behavior of the contributions to $R$, which were obtained from the perturbation theory for $S_J$ and $S_H$. In the first case ($S_J$), after a direct integration of the renormalization-group equations, we see that the series expansion is carried out over the effective coupling constant $g$:

$$g = (E_J \tau_s)(T\tau_s)^{\alpha_s^{-1}-1}; \qquad (\alpha_s = 2\pi\eta_s; \tau_s = CR_s, \quad E_J = I_c/2e)$$

and $g \to 0$ when $T \to 0$ if $\alpha_s < 1$. In the second case a lowering of the temperature enhances the role of the nonlinear effects and the only remaining small parameter of the perturbation theory is the quantity $\gamma = \eta_0/\eta$, $(\eta = \eta_0 + \eta_s)$. More exactly, the condition $\gamma \ll 1$ becomes a necessary condition if $(E_Q/T) \gg 1$, where $E_Q = e^2/2C$ is the Coulomb energy which is related to the electron tunneling through the junction.

Calculating $Z(\xi I)$ in the first nonvanishing order of the perturbation theory in $S_J$ and $S_H$, we find from Eq. (4) the dynamic resistance

$$R(R_s^{-1} + R_0^{-1}) = 1 - \mu_1 + \mu_2 \,, \tag{5}$$

$$\mu_2 = \frac{\gamma}{a} \int_0^\infty \frac{dx}{x} f^2\left(\frac{\pi x}{2a}\right) \sin\left[\frac{\pi}{4\alpha}(1 - e^{-x})\right] \exp\left(-\frac{h(x)}{4}\right), \tag{6}$$

$$h(x) = \frac{2}{\alpha} \int_0^\infty \frac{dz}{z} (1 - \cos xz)(z^2 + 1)^{-1} \coth za, \tag{7}$$

where

$$a = (2T\tau_0)^{-1}; \quad \tau_0 = C(R_0^{-1} + R_s^{-1})^{-1}; \quad \alpha = 2\pi\eta; \quad f(x) = x/\sinh x.$$

After the substitution $\eta_s \to \eta$, the expression for $\mu_1$ will be the same as the result of Refs. 3 and 6 and therefore will not be given here. We must emphasize, however, that this expression is proportional, as was pointed out above, to the square of the effective coupling constant $g^2[\mu_1 \to 0$ if $T \to 0$ $(\alpha < 1)]$. The quantity $\mu_2$ is a new contribution resulting from the fact that $S_H$ is nonquadratic in $\phi$ and $\chi$.

Equations (6) and (7) can be slightly simplified by using, as was done in Ref. 3, an exponential function $e^{-z}$, instead of the function $(z^2+1)^{-1}$, in the integral in (7)

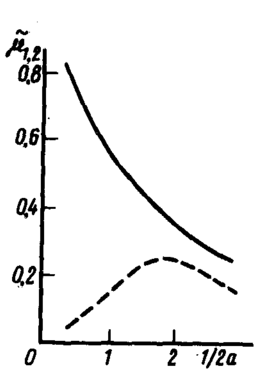


FIG. 1. Dependence of $\tilde{\mu}_1 = \mu_1 | (E_J \tau_0)^2$ (dashed curve) and $\tilde{\mu}_2 = \mu_2/\gamma$ (solid curve) on $(1/2a) = T\tau_0$, $(\alpha = 1/4)$.

and the argument of sin which is written as an integral. As a result, we find

$$\mu_2 = \frac{2}{\pi}\gamma \int_0^\infty \frac{dx}{x} f^2\left(\frac{\pi x}{2a}\right)\sin\left(\frac{1}{2\alpha}\arctan x\right)\left(1+x^2\right)^{-\frac{1}{4\alpha}} \left|\frac{\Gamma\left(1+\frac{1}{2a}+\frac{ix}{2a}\right)}{\Gamma\left(1+\frac{1}{2a}\right)}\right|^{1/\alpha} \tag{8}$$

In the limit $a \gg 1$, we find from (8) $\mu_2 \simeq \gamma$; at low temperatures the perturbation theory is applicable only at $\gamma \ll 1$. If $a\alpha \ll 1$, we can use the asymptotic expansion $\Gamma(x)$. As a result, we can write (8) as

$$\mu_2 = \frac{2}{\pi}\gamma \int_0^\infty \frac{dx}{x} f^2\left(\frac{\pi x}{2a}\right)\left(\sin\frac{x}{2\alpha}\right)(1+x^2)^{1/4a\alpha_0} e^{-\frac{x}{2a\alpha}\arctan x} \tag{9}$$

In the limit $a\alpha \ll 1$, the result of integration of (9) depends on the relation $(a/\alpha) = (1/\pi)(E_Q/T)$. At $E_Q/T \ll 1$ we find $\mu_2 = (2\gamma/3\pi)(E_Q/T)$ and in the opposite case, $E_Q/T \gg 1$, we find[1)] $\mu_2 \simeq \gamma$.

In the low-viscosity region, $\alpha \ll 1$, $a\alpha \ll 1$, a simple expression can be obtained for any value of $a$ from the exact equations [Eqs. (6) and (7)].

Using the explicit expression for the Fourier transform $\tilde{\alpha}(\omega)$, we find

$$\mu_2 = \frac{\gamma}{2\sqrt{\pi}}\left(\frac{E_Q}{T}\right)^{3/2} \int_{-\infty}^{\infty} dz\, z(z+1)\coth\frac{(z+1)E_Q}{2T}\exp\left(-\frac{E_Q}{4T}z^2\right). \tag{10}$$

The limiting cases given here are derived from Eq. (10). (In the limit $E_Q/T \ll 1$, only the factor $2/\pi$ is missing in the $\mu_2$.) Furthermore, its structure resembles the expression for the current correlator in the shot noise.[11] This correlator is proportional to $\delta^2 Z/\delta\xi_{-\omega}\delta\xi_\omega$ and it reproduces the equation in Ref. 11 in the limit $I = 0$, $\alpha \ll 1$. Using the generating functional $Z$, we can calculate the average (with respect to time) voltage across the junction:

$$2(e\bar{V} - \bar{\omega}) = -\operatorname{sign}I \int_0^\infty d\tau \left\{ \frac{2\gamma}{\eta_0}\alpha_0(\tau)\sin|\bar{\omega}|\tau \sin\left[\frac{\pi}{4\alpha}(1 - e^{-(\tau/\tau_0)})\right] e^{-(1/4)h(\tau/\tau_0)} \right.$$

$$\left. + (2\pi/\alpha)E_J^2 \sin(2|\bar{\omega}|\tau)\sin\left[\frac{\pi}{\alpha}(1 - e^{-\tau/\tau_0})\right]\exp(-h(\tau/\tau_0)) \right\}, \tag{11}$$

where $\bar{\omega} = I/4e\eta$ ($I$ is the constant current). In the limit $\alpha \ll 1$, $a\alpha \ll 1$, the right side of (11) changes to

$$\frac{\operatorname{sign}I}{4\eta\sqrt{\pi T}}\left[E_Q^{3/2}\int_{-\infty}^{\infty} dz\, e^{-(z^2E_Q)/4T}\left(\tilde{\alpha}\left(z+1+\frac{|\bar{\omega}|}{E_Q}\right) - \tilde{\alpha}\left(z+1-\frac{|\bar{\omega}|}{E_Q}\right)\right)\right.$$

$$\left. - \frac{\pi E_J^2}{\sqrt{E_Q}}\sinh\frac{|\bar{\omega}|}{T}\exp\left(-\frac{E_Q}{T} - \frac{\bar{\omega}^2}{4TE_Q}\right)\right] \tag{11'}$$

For $\overline{\omega}/E_Q \gg 1$ and $E_J = 0$ we derive from this expression

$$V = \frac{\bar{\omega}}{e} + \gamma \frac{e}{2C} \operatorname{sign} I \,. \tag{12}$$

In the limit $\overline{\omega}/E_Q \ll 1$, $E_Q/T \gg 1$, Eqs. (11) and (11′) are valid if $\gamma \ll 1$. Setting $\gamma = 1$ in (12), we obtain the result of Ref. 4.

The crossover temperature $T^*$, which corresponds to the transition from a regime where $R$ decreases to one where $R$ increases as a function of the temperature (if $\alpha < 1$), was found in Ref. 3 by means of Eq. (5) (without the term $\mu_2$). $T^*$ depends on $\alpha$ ($T^* \to 0$ as $\alpha \to 1$). If $\alpha \ll 1$, then $T^* \approx (2/3)(E_Q/T)$ (Ref. 3). At $\alpha \ll 1$ the nonlinear effects are thus appreciable all the way to the crossover temperature, which now must be determined with allowance for $\mu_2$.

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[1)]A numerical calculation of $(1/\gamma)\mu_2$ and $(E_J\tau_0)^{-2}\mu_1$ for $\alpha = 1/4$ is shown in Fig. 1.

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