Transition from harmonic behavior to chaos in the interference of plane waves in a nonlinear medium

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Experiments and numerical simulations on the interference of plane waves described by a one-dimensional nonlinear Schrödinger equation show that a transition can occur from a simple harmonic behavior of the system to a dynamic chaos.

There are many examples of the appearance of a dynamic chaos in the behavior of physical systems. One well-known and striking event is the transition from laminar to turbulent flow in hydrodynamics. In optics, there are fewer systems in which a transition from a simple (periodic) behavior to a complex (random) behavior has been predicted theoretically. These cases are the transitions to chaos in ring resonators,[1,2] in bistable optical devices such as nonlinear Fabry-Perot resonators,[3,4] and in second-harmonic generation.[5] Experimentally, we have only the observation of a chaos accompanying optical bistability,[6] a study of the onset of "turbulence" in the frequency spectrum during the propagation of electromagnetic waves in a waveguide with a nonlinear conductivity,[7] and a study of a transition of dynamic chaos in the spectrum of oscillations in the output power of a ring gas laser.[8] In one way or another, these examples involve an external feedback, although this is not a necessary condition, as was shown in Ref. 9.

In this letter we wish to demonstrate a transition to dynamic chaos in the interference of plane waves in media with a nonlinear refractive index.

Two coherent plane light waves of equal intensity are propagating through a nonlinear medium at angles $\pm\theta$ with respect to the longitudinal (z) axis. We examine the behavior of light rays by working from a solution of the well-known parabolic equation for the electric field E of an electromagnetic wave which is propagating along the z axis:

$$2ik\frac{\partial E}{\partial z}+\frac{\partial^2 E}{\partial x^2}-k^2\frac{\epsilon_2|E|^2}{\epsilon_0}E=0, \tag{1}$$

where $k=2\pi n/\lambda_0$ and $\epsilon=\epsilon_0-\epsilon_2|E|^2$ is the complex dielectric constant of the medium.

Equation (1) is a nonlinear one-dimensional Schrödinger equation. The formulation of the problem as in (1) allows us to introduce a new dynamic variable—the coordinate z—which has not been used in previous papers.

The transverse distribution of the field at the entrance surface is $E(x,0)=E_0\cos(2\pi x/\Lambda)$, where $\Lambda=\lambda/2\theta$ (for small values of θ) is the period of the interference pattern. The interference of plane light waves in a nonlinear medium gives rise

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to transverse spatial harmonics of the field. A superposition of these harmonics, $E = \Sigma_{j=0}^{\infty} E_j(z)\cos[(2j+1)k\theta x]$, is a solution of (1). For small nonlinearities, with $E_j \gg E_{j+1}$, solutions found by perturbation theory can be used:

$$|E_1/E_0| = \frac{\mu}{16}\left|\sin\left(\frac{4z}{L}\right)\right| \tag{2}$$

and

$$|E_2/E_0| = \frac{\mu^2}{1536}\sin^2\left(\frac{4z}{L}\right)\sqrt{8\cos^2\left(\frac{4z}{L}\right)+1}\ , \tag{3}$$

where $L = 2/k\theta^2$ represents the diffraction length of a beam with a transverse dimension Λ, and $\mu = (\epsilon_2/\epsilon_0\theta^2)|E_0|^2$.

It follows from (2) and (3) that in the case of a small nonlinearity, the intensity of the spatial harmonics is an undamped (if there is no absorption) periodic function of z. The period of the intensity modulation for the first spatial harmonic is given by the expression $z_0 = \lambda/4\theta^2$. We find the same periodicity in the solution for E_1 for any negative nonlinearity of the type $\delta\epsilon \sim |E|^{2m}$, where m is an integer.

For the case of high intensities, we have solved Eq. (1) numerically. The results of the numerical calculations on the propagation of interfering plane waves in a nonlinear medium show that at low incident intensities we typically see a harmonic behavior of the intensities of the spatial harmonics, in agreement with solutions (2) and (3), but as the intensity increases (Fig. 1), the behavior of the field begins to change substantially, and there is a doubling of the period of the variations in the intensities of the spatial harmonics along the z axis. As the intensity is increased further, the behavior of the spatial harmonics along the propagation direction becomes even more com-

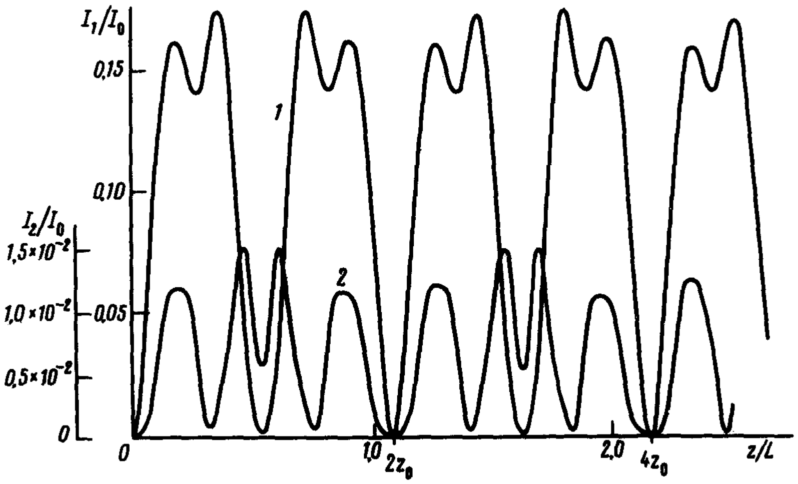

FIG. 1. Relative intensities of the first (1) and second (2) spatial harmonics with $\mu = 17$.

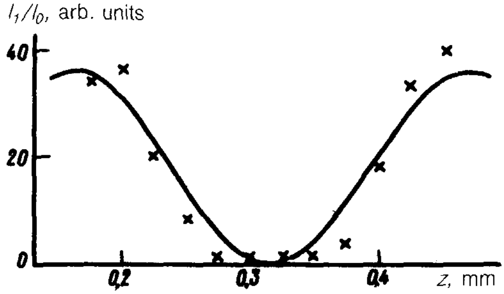

FIG. 2. Relative intensity of the first spatial harmonic during the interference of waves in silicon with $\mu \sim 1$ ($I_{inc} \sim 4$ MW/cm^2; $2\theta_0 = 0.104$ rad).

plicated and undergoes a transition to a dynamic chaos, where the coordinate z serves as the dynamic variable.

Although we do find a period doubling (Fig. 1), we do not have an adequate basis to assert that the transition to chaos occurs strictly in accordance with Feigenbaum's model.[10] It is possible that there are also other pathways to a dynamic chaos.

To verify the harmonic behavior of spatial harmonics (2) along z, we carried out an experiment. The beam from a Nd:YAG laser (single-frequency, TEM_{00} mode, pulse length of 10 ns, energy of 1–10 mJ in a pulse) is split into two beams of equal intensity by means of interference mirrors. These two beams cross at an angle $2\theta_0 = 0.104$ in a nonlinear medium: a sample of silicon or gallium arsenide. The samples are wedge-shaped plates with a wedge angle of 16′, so that the thickness of the medium can be varied by moving the wedge. We measure the intensity of the first spatial harmonic which is propagating in the direction of the first diffraction maximum at various thicknesses of the nonlinear medium. The light in the first diffraction maximum is spatially separated and sent to a photodetector by a lens and a lightguide cable.

Figure 2 shows some experimental results for low incident intensities ($\sim$4 MW/cm^2), along with a calculated curve.

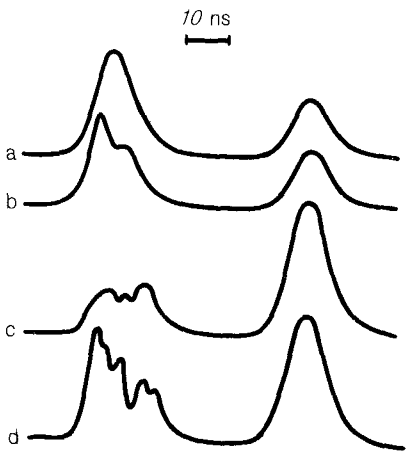

FIG. 3. Oscilloscope traces of the light diffracted into the first maximum (the first pulse on the traces) and of the incident light for several intensities: a— $\sim$10 MW/cm^2; b— $\sim$30 MW/cm^2; c, d— $\sim$60 MW/cm^2.

In addition, we fixed $z(z=0.3$ mm) and measured the time evolution of the intensity of the diffracted pulse. These measurements made it possible to study the behavior of the intensity of the spatial harmonic during the pulse at various intensity levels. Gallium arsenide, in which the recombination time for nonequilibrium charge carriers is $\sim 10^{-9}$–10^{-10} s, is a suitable model here, since the magnitude of the nonlinearity in the medium at each instant is determined by the intensity of the incident light; i.e., the agent acting on the medium is in a steady state. The onset of chaos in the behavior of the spatial harmonic at a fixed z, on the other hand, corresponds to an analogous behavior over time at intensities typical of the onset of chaos. Figure 3 shows some oscilloscope traces of pulses diffracted into the first maximum and of the incident light. We see that the behavior of the intensity in the first diffraction maximum in Fig. 3, c and d, is random at high excitation levels. The possibility of an onset of chaos should be kept in mind in interpreting experiments on photoinduced self-diffraction.

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