Possibility of MHD-stable plasma confinement in an axisymmetric mirror system

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If the magnetic field of an axisymmetric plasma confinement system is tailored appropriately, it can keep the plasma stable with respect to all flute modes.

An active search is being made for axisymmetric open confinement systems in which a plasma would be stable with respect to flute modes. Although many schemes taking a variety of physical approaches have been proposed,[1–8] all are afflicted with shortcomings of one sort or other, so that the problem cannot be regarded as solved. In the present letter we show that it is possible—without going beyond the scope of the traditional geometry of a mirror system and without appealing to any new physical stabilization mechanisms—to achieve absolutely stable (with respect to all flute modes) plasma confinement in an axisymmetric open system. The approach proposed here is based on the choice of a special configuration for the magnetic field of the mirror system. For brevity, we restrict the analysis to the case of a plasma with a sharp boundary, in which case the pressure, varying smoothly from the axis of the confinement system out to the edge, drops sharply to zero at the boundary line of force. We assume $\beta \ll 1$ [$\beta = 8\pi(p_\perp + p_\parallel)/B^2$, where $p_\parallel$ and $p_\perp$ are the longitudinal and transverse components of the plasma pressure, and B is the magnetic field]. We can then write the condition for stability of the plasma boundary with respect to flute waves as[9]

$$I = \int \frac{\kappa \, ds}{r(s) B^2(s)} (p_\perp + p_\parallel) > 0, \tag{1}$$

where κ is the curvature, and $r(s)$ is the instantaneous radius of the boundary line of force. The integration in (1) is along this line from mirror to mirror. We consider a mirror system with a mirror ratio which is only slightly different from unity: $\epsilon \equiv B_{\max}/B_{\min} - 1 \ll 1$. The magnetic field of such a mirror system can be written

$$\mathbf{B} = \mathbf{B}_0 + \mathbf{b}(r, z), \tag{2}$$

where $\mathbf{B}_0$ corresponds to a uniform magnetic field which is directed along the z axis, and $\mathbf{b}$ specifies an axisymmetric mirror configuration. In such a mirror system the lines of force are nearly straight; the longitudinal pressure is small in comparison with the transverse pressure ($p_\parallel \sim \epsilon p_\perp$); and we have $\kappa \simeq B_0^{-1} \partial b_r/\partial z$. As a result, we can replace (1) by

$$I = \frac{1}{rB_0^3} \int p_\perp \frac{\partial b_r}{\partial z} dz . \tag{3}$$

The sign of I depends on the pressure distribution along a line of force. We assume

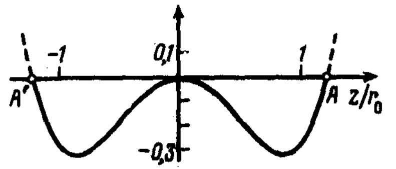

FIG. 1. Line of force in a system of two turns of radius R, positioned at the points $z = \pm R/2$. Plotted along the ordinate is the difference $r - r_0$ divided by r_0^5/R^4. The magnetic field increases from the point $z = 0$ to the ends of the solid line (points A' and A), where it begins to decrease (the dashed line).

that this distribution is step-shaped: $p_\perp$ remains constant all the way to points where the field reaches a maximum, at which this pressure component vanishes. We can then write

$$I = \frac{2p_\perp}{rB_0^3} b_r^{(1)}, \tag{4}$$

where $b_r^{(1)}$ is the radial component of the magnetic field at the point of a maximum of the field modulus on a line of force which lies to the right of the $z = 0$ plane. We assume that this plane is the symmetry plane of the mirror system. It is customary to assume that at a mirror the lines of force run parallel to the axis of the confinement system. That picture, however, is valid only for the paraxial field, and then only in the first approximation with respect to the paraxial nature of the configuration. Figure 1 shows an example of a line of force of a magnetic field for which the relation $b_r^{(1)} > 0$ holds. This field is produced by two thin circular turns of radius R, which are at the distance $d = R$ and which carry identical currents. Figure 1 shows a line of force which intersects the $z = 0$ plane at a distance r_0 from the axis, where $r_0 \ll R$. The positive slope of the curve at point A corresponds to the relation $b_r^{(1)} > 0$.

In the example shown in Fig. 1, the mirror ratio tends toward unity as the axis of the system is approached (it can be shown that we have $\epsilon = 2.6 r_0^4/R^4$), so that the plasma is not confined at the axial line of force. This shortcoming can be corrected easily by slightly increasing the distance d, to satisfy the condition $d > R$, while maintaining $d - R \ll R$. In this case, two maxima of B arise at the axis, while at $R \gg r_0 \gg \sqrt{R(d-R)}$ the shape of a line of force is no different from that in the case $d = R$.

A more detailed analysis shows that the presence of a sharp plasma boundary is not a necessary condition for stability. There is a fairly wide class of stable profiles in which $p_\perp$ varies continuously both in the radial direction and along a line of force. An important point here is that in such profiles the pressure falls off monotonically with distance from the center of the confinement system to a mirror (vanishing at the point of a field maximum). In this regard, the method proposed here is fundamentally different from the method of plasma stabilization by means of "sloshing ions," in which case stability is achieved by producing a pressure peak near the mirrors in a region of favorable curvature.

We note in conclusion that the requirement of a small mirror ratio which we used above is not of fundamental importance for this method. It can be shown that there exist axisymmetric magnetic field configurations with $\epsilon_r \gtrsim 1$ in which MHD-stable plasma confinement can be achieved.

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