Equation of continuity for the helicity in media with an infinite conductivity

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(Submitted 10 January 1987)
Pis'ma Zh. Eksp. Teor. Fiz. **45**, No. 5, 215–216 (10 March 1987)

The magnetic field helicity density is shown to satisfy the equation of continuity in the case of a special vector potential gauge. A new functional series of conserved integrals over the volume of an infinitely conducting medium is found.

The equations for the electromagnetic field in media with an infinite electrical conductivity but a finite current density are relativistically invariant:

$$\partial_t \mathbf{B} = -c \operatorname{curl} \mathbf{E}; \quad \mathbf{E} = [\mathbf{B}, \mathbf{v}]/c. \tag{1}$$

Here $\mathbf{v}$ is the velocity of the medium which below is assumed to be an arbitrary function of time and the coordinates. Equation (1) implies the freezing-in theorem on the conservation of the magnetic field flux through an arbitrary closed circuit whose points move at the velocity $\mathbf{v}$ (a fluid circuit).

We derive from (1) the equation of continuity which implies the conservation laws in liquid volume. For this purpose, we introduce the vector potential $\mathbf{A}$, which is calibrated as follows:

$$\mathbf{B} = \operatorname{curl} \mathbf{A}, \quad c\mathbf{E} = -\partial_t \mathbf{A} - \nabla (\mathbf{A}\mathbf{v}). \tag{2}$$

It is easy to show that for any solution of (1), including $\mathbf{E} = -\nabla\varphi$, the vector

 0021-3640/87/050266-02$01.00

function **A** is defined within the addition $\nabla\psi$, where ψ is an arbitrary solution of the equation $\partial_t \psi + \mathbf{v}\nabla\psi = \text{const}$.

We introduce a pseudoscalar—the helicity density $s = \mathbf{AB}$. It follows from (1) and (2) that s satisfies the equation of continuity:

$$\partial_t s + \operatorname{div}(s\mathbf{v}) = 0. \tag{3}$$

If the density ρ and the pressure p satisfy the equations

$$\partial_t \rho + \operatorname{div}(\rho \mathbf{v}) = 0; \quad \partial_t p + \mathbf{v}\nabla p = -\gamma p \operatorname{div} \mathbf{v}, \tag{4}$$

we can write the functional series of the first integrals of (3) and (4):

$$I = \int_V s f(\rho/s, p^{1/\gamma}/s)\, d\mathbf{r}, \tag{5}$$

where f is an arbitrary function of two variables, and V is an arbitrary volume whose points move at a velocity $\mathbf{v}$.

At $f \equiv 1$ the quantity V, the total volume in (5), changes to a helicity integral which was found in Ref. 1 and which was used in Refs. 2 and 3 to analyze the stability of plasma force-free configurations ($p = \text{const}$). Integral (5) can be used to study the flow and structure stability when $\nabla p \neq 0$.

We wish to thank V. D. Shafranov for a stimulating discussion.

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Translated by S. J. Amoretty