Mechanism for adjusting the rotational-transform profile in a stellarator

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The basic features of the changes caused in the profile of the rotational transform in a stellarator by a finite plasma pressure are explained.

The rotational transform is one of the most important characteristics of a stellarator, largely determining its confinement properties. The behavior of the vacuum rotational transform, μ_h, which is created by the external helical fields, is well known for each particular device. In the most general case we would have

$$\mu_h = \mu_0 + (\mu_b - \mu_0)\rho^2/b^2 \quad , \tag{1}$$

where μ_0 is the value of μ_h at the geometric axis of the stellarator, ρ is the radius, reckoned from this axis, and $\mu_b = \mu_h(b)$. Expression (1) covers all existing and planned stellarators either without shear or with a large shear, except for Heliotron-*E*, in which the relation $\mu_h = 0.55 + 1.95\rho^4/b^4$ holds.

As the plasma pressure is raised, the rotational transform changes, in such a way that at values of β (the ratio of the plasma pressure to the magnetic pressure) near the equilibrium limit β_{eq} the rotational transform loses the basic positive qualities which it had at $\beta = 0$: a monotonic behavior in stellarators with shear and the absence of resonant values of m/n with small values of m and n in stellarators without shear. This fact, which has been solidly established in numerical calculations, is presently attracting particular interest, since the capabilities of existing stellarators would make it possible to approach β_{eq} even today. Further interest in these effects is inspired by their seemingly puzzling nature and their unpredictability.[1–3] From the theoretical standpoint, these effects are of fundamental importance in stability problems.[1,3,4]

The rotational transform in a stellarator, μ_{st}, which is equal to μ_h at $\beta = 0$, is determined in the absence of a longitudinal current by exclusively the configuration of the magnetic surfaces. It is thus logical to seek the reason for the distortion of the μ_{st} profile with increasing β in a change in the internal configuration of the plasma column.

A basic effect of a finite plasma pressure in a toroidal system is an outward displacement of the magnetic surfaces (a ballooning). Furthermore, in the limit $\beta \to \beta_{eq}$ in a current-free stellarator, the inner surfaces are stretched out noticeably (becoming elliptical), even if the plasma boundary is circular. The simplest parametric specification of the average magnetic surfaces, $a(\mathbf{r}) = \text{const}$, which allows us to explicitly incorporate their displacement Δ and elongation K is

$$\rho\cos u = a\cos\theta - \Delta, \qquad \rho\sin u = K(a)a\sin\theta \, , \tag{2}$$

where ρ and u are quasicylindrical coordinates which are related to the circular geometric axis. The function μ_{st} is related to K and Δ through the metric coefficients of the flux coordinate system with straightened lines of force, which can be calculated for given $\rho(a,\theta)$ and $u(a,\theta)$ from the general equations in Ref. 5. Using them, we find that for stellarators with μ_h as in (1) we would have

$$\mu_{st} = \frac{2K}{K^2+1}\left[\mu_0 + \frac{\mu_b - \mu_0}{b^2}\left(a^2\frac{K^2+1}{2} + 2\Delta^2\right)\right](1-\delta), \tag{3}$$

where the quantity δ, which is small in comparison with unity, is given by

$$\delta = \frac{1}{2(1+d)^2}\left[\Delta'^2 + d^2 + \frac{K^2-1}{K^2+1}\left(d^2 + d + \frac{\Delta'^2}{2}\right)\right]. \tag{4}$$

Here $d \equiv aK'/(2K)$; and the prime means the derivative with respect to a. In deriving (3) we assumed $|\Delta'| \ll 1$ and $|d| \ll 1$.

Expression (3) can explain many aspects of the behavior of μ_{st} which have appeared puzzling. We begin with systems without shear ($\mu_b = \mu_0$).

In stellarators of this type, the main cause of a change in μ_{st} with increasing β is the shift Δ. The graphic formula

$$\mu_{st} = \mu_0(1 - \Delta'^2/2), \tag{5}$$

which holds for $k = 1$ (circular surfaces), shows that the displacement of the surfaces leads to a decrease in μ_{st} everywhere except at the magnetic axis, where we have $\Delta' = 0$. The radial profile of μ_{st} depends on $\Delta'(a)$. We can find an explicit relationship between $\Delta'(a)$ and the pressure $p(a)$ by working from the equilibrium equations (Ref. 5, for example):

$$\Delta' = \frac{R}{\mu_0^2}\,\frac{2[\bar{p}(a) - p(a)]}{aB_0^2}; \qquad \bar{p}(a) \equiv \frac{2}{a^2}\int_0^a p(a)a\,da. \tag{6}$$

Here R is the radius of the geometric axis, and B_0 is the longitudinal field. The pressure distribution $p = p_0(1 - a^2/b^2)^2$ corresponds to

$$\Delta' \doteq \frac{2p_0}{B_0^2}\,\frac{R}{\mu_0^2 b}\left(\frac{a}{b} - \frac{2}{3}\frac{a^3}{b^3}\right). \tag{7}$$

The profile $\mu(a)$ in (5) corresponding to this functional dependence $\Delta'(a)$ is shown in Fig. 1: $\mu(a)$ decreases at $a/b < \sqrt{0.5} \approx 0.7$ and increases at $a/b > 0.7$. This behavior of $\mu_{st}(a)$, which was found in numerical equilibrium calculations,[1–3] has not previously been explained. It can be seen from (5) and (6) that the functional dependence $\mu_{st}(a)$ becomes nonmonotonic only if the pressure profiles have peaks. With $p = p_0(1 - a^2/b^2)$, the profile $\mu_{st}(a)$ is parabolic:

$$\mu_{st}(a) = \mu_0\left[1 - \frac{1}{8}\,\frac{\beta_0^2}{\beta_{eq}^2}\,\frac{a^2}{b^2}\right]. \tag{8}$$

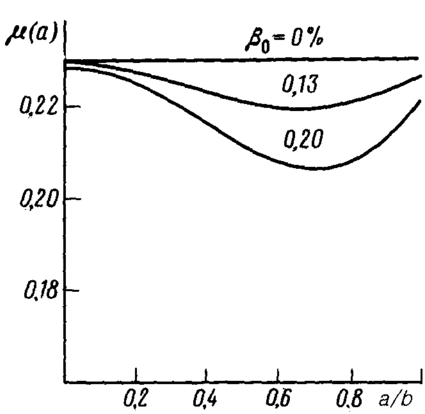

FIG. 1. Rotational transform in a stellarator without shear with $\mu_0 = 0.23$, $R/b = 20$, and $p = p_0(1 - a^2/b^2)^2$.

Here $\beta_0 = 2p_0/B_0^2$ and $\beta_{eq} = \mu_0^2 b/R$. These comments also apply to configurations with noncircular surfaces. Among the new features in the behavior of μ_{st} caused by the prolate nature of the surfaces ($K > 1$) in a stellarator without shear is a small overall decrease in μ_{st} [see (3)], which is more pronounced at the center because of the slight increase in K toward the center (Fig. 2).

Expression (3) shows that in stellarators with a large vacuum shear the effect of the elliptic shape on μ_{st} is far greater than in the case $\mu_b = \mu_0$. This effect is directly opposite the effect which a displacement has on μ_{st} (Ref. 5): The elongation of the magnetic surfaces leads to a decrease in $\mu_{st}(0)$ by a factor of $2K/(K^2 + 1)$ and to an increase in μ_{st} at the edge of the plasma column [the term $K(\mu_b - \mu_0)a^2/b^2$] at $K > 1$. The undesirable deformations of $\mu_{st}(a)$ which result from the toroidal displacement are effectively suppressed here, even if the elongation of the boundary is only moderate (Fig. 3). The possibility of controlling the $\mu_{st}(a)$ profile in the ATF stellarator was first studied numerically in Ref. 6. Our analysis reveals the mechanism for this control and demonstrates that the elongation plays a governing role. It is important to note

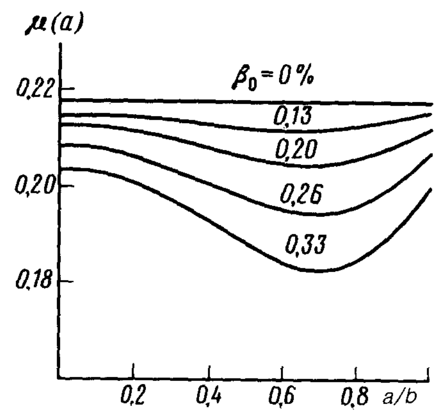

FIG. 2. Calculated profile $\mu_{st}(a)$ in a stellarator without shear ($\mu_0 = 0.23$, $R/b = 20$) with a noncircular plasma boundary. $K(b) = 1.4$; $p = p_0(1 - a^2/b^2)^2$.

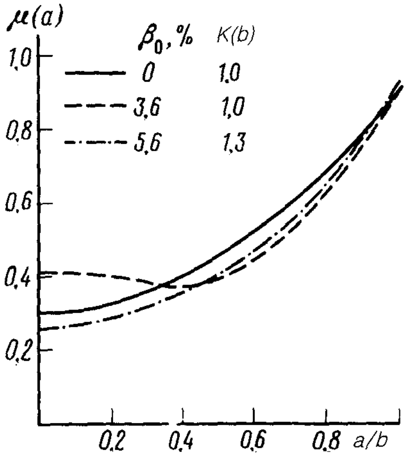

FIG. 3. Rotational transform in a stellarator with shear ($R/b = 7$).

that $K(a)$ increases sharply toward the center in a stellarator with shear, intensifying the favorable effect of the prolate shape on μ_{st}.

The problem of finding $\mu_{st}(a)$ is closely related to the equilibrium problem (calculating Δ and K). To solve this problem, we are using the EMEQ-ST modification of the EMEQ code[7] for a tokamak. A knowledge of Δ and K is sufficient for reliably describing the profiles $\mu_{st}(a)$ found by accurate calculations (Figs. 2 and 3).

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