Influence of localization and correlation effects in the scattering on the conductivity in the ultraquantum limit

S. S. Murzin
Institute of Solid State Physics, Academy of Sciences of the USSR

(Submitted 25 December 1986)
Pis'ma Zh. Eksp. Teor. Fiz. **45**, No. 5, 228–231 (10 March 1987)

The motion of an electron in a degenerate semiconductor in a strong magnetic field is analyzed on the basis of a semiclassical description. The localization and correlation in the scattering are taken into account. The conductivity along the magnetic field $\sigma_{\parallel}$ and perpendicular to it $\sigma_{\perp}$ at low temperature is determined.

Only the effect of the magnetic field **H** on the motion trajectories, the spectrum, and the characteristics of the mutually independent electron scattering events have until recently been taken into account in the study of the conductivity of semimetals and semiconductors in the ultraquantum limit.[1,2] It has recently become clear that since the motion of electrons is nearly one-dimensional in the ultraquantum limit, the localization (interference) effects must also be taken into account.[3] Furthermore, it follows from Refs. 4–6 that if the magnetic length λ is much shorter than the screening length d of the impurities, all scattering events cannot be assumed to be independent. Correlations arise because the electron returns many times to the field of the same

 0021-3640/87/050283-05$01.00

impurity, shifting slightly across **H**. Moving in the field of this impurity, it will drift each time in the same direction until it shifts a distance of $\sim d$.

In this letter we derive an expression for $\sigma_\parallel$ of a degenerate uncompensated or slightly compensated semiconductor in an ultraquantum limit when the localization effects are important but the temperature $T \neq 0$ (the restrictions imposed on T are given below). An equation which establishes a relationship between $\sigma_\perp$ and $\sigma_\parallel$ and which takes into account the multiple returns of the electron into the field of the same impurity is $\sigma_\perp \propto \sigma_\parallel^{-1/3}$, and $\sigma_\perp$ is determined from the known $\sigma_\parallel$. Note that the relationship between $\sigma_\perp$ and $\sigma_\parallel$ is of a more general nature than the expressions for $\sigma_\perp$ and $\sigma_\parallel$. Specifically, this relation is valid in the classical case, which is considered in Ref. 5 and in Secs. B and D of Ref. 6.

We assume that $k_z \lambda \ll 1$ and $k_z^2 \hbar^2/2m > e^2/\epsilon N^{-1/3}$, where k_z is the wave vector of an electron along **H**, m is the effective mass, N is the density of the ionized impurities, and ϵ is the dielectric constant of the crystal lattice. It follows from these inequalities that $\lambda \ll d$. Let us consider the motion of an electron on different time scales.

1. During the time τ_1—the time of travel of the electron before it is backscattered, the electron is scattered across the magnetic field, as it moves along **H**, by many randomly situated impurities. The transverse motion is therefore diffusive in nature during this time, i.e., the transverse displacement is $|\rho| \propto t^{1/2}$. The total displacement during the time τ_1 within the logarithmic factors is

$$\rho_1 \sim (D_\parallel \tau_1)^{1/2} \sim \lambda^2 \left(k_z^2 + \frac{1}{4} d^{-2}\right)^{1/2} << \lambda \ . \qquad (1)$$

In other words, it is small in comparison with the transverse dimension of the wave function of the electron. The diffusion coefficient D and τ_1 were found in Ref. 1.

2. If there were no transverse motion at all, the electron would also be localized[7] along the field direction because of the one-dimensional nature of the motion. The one-dimensional localization is destroyed by the motion across **H**. However, since the displacement of the electron across **H** is only slight during the time τ_1, the localization effects have a strong effect on $\sigma_\parallel$ at $T=0$, as shown in Ref. 3.

The result of Ref. 3 can be obtained by means of the following qualitative considerations. Since the localization length is $\sim l = \hbar k_z \tau_1/m$ in the one-dimensional case, we can treat the electron as a wave packet of length $\sim l$ and transverse dimension λ. The transverse component of the nonuniform electric field of the impurities will displace such a wave packet across the field **H**. At scale dimensions smaller than d, the transverse motion of the packet is similar to the drift with a velocity[1)]

$$v_x = \frac{c}{H} \int \psi^*(\mathbf{r}) E_y(\mathbf{r})\, \psi(\mathbf{r})\, d\mathbf{r} = \int_{-\infty}^{\infty} P(z) E_y(z)\, dz \sim c\, \frac{E_d}{H} P_1 d \cdot N_l^{1/2} \sim \frac{ceN^{1/2}}{\epsilon H l^{1/2}} \ , \qquad (2)$$

where $\psi(r)$ is the electron wave function,

$$E_y = \sum_i \frac{e y_i}{\epsilon (r - r_i)^3}\, e^{-\frac{|r - r_i|}{d}}$$

is the electric field of the impurities, $P(z)dz$ is the probability that the electron would be found in the interval dz, $E_d \sim e/\epsilon d^2$, $P_1 \sim l^{-1}$, and $N_l \sim Nld^2$ are the impurities with which the wave packet interacts simultaneously.

At a temperature $T = 0$ the electron moves a distance of $\sim l$ along the direction of the field **H**, whereas the transverse motion of the electron causes the localization conditions to change; i.e., the localization conditions change when the electron is displaced across the field **H** a distance of $\rho_{20} \sim (k_z^2 + 1/4d^{-2})^{-1/2}$. The corresponding time, within the logarithmic factors, is

$$\tau_{20} \sim \frac{\rho_{20}}{v_x} \sim \frac{\epsilon H l^{1/2}}{ce N^{1/2}\left(k_z^2 + \frac{1}{4} d^{-2}\right)^{1/2}} \sim \tau_1 \lambda^{-2}\left(k_z^2 + \frac{1}{4} d^{-2}\right)^{-1} >> \tau_1. \quad (3)$$

The diffusion in the direction of **H**

$$D \sim l^2/\tau_{20} \sim (l^2/\tau_1)\lambda^2\left(k_z^2 + \frac{1}{4} d^{-2}\right), \quad (4)$$

and hence the conductivity, are a factor of $\lambda^{-2}(k_z^2 + 1/4d^{-2})^{-1}$ lower than the values found in the standard kinetic theory.[1] In the case $k_z \gg d^{-1}$, which was considered in Ref. 3, this result coincides, within the logarithmic factors, with the result of Ref. 3. It is not clear, however, whether this result can be regarded as definitive. Consequently, we will consider only the case $T \neq 0$, where the inelastic scattering time is $\tau_1 \ll \tau_{in} \ll \tau_{20}$.

The time τ_{in} is the second scale time of interest to us. During the time τ_{in}, the electron is nearly localized along the field **H** but drifts across it. At $t > \tau_{in}$ and $kT > \hbar/\tau_1$ the electron moves along the field direction in a series of jumps of length $\sim l$ and frequency τ_{in}^{-1}. The diffusion is

$$D_{\parallel} \sim l^2/\tau_{in}. \quad (5)$$

Using the Einstein relation, we find the following expression for the conductivity:

$$\sigma_{\parallel} \sim \frac{ne^2\tau_1}{m}\frac{\tau_1}{\tau_{in}}. \quad (6)$$

3. The drift nature of the motion at $t < \tau_{in}$ may be assumed to occur as a result of the correlation in the scattering across the field **H**. The electron, whose motion along **H** is restricted by the region of length l, moves through the field of each impurity in this region many times and drifts each time in the same direction in this field.

The scattering correlations should also be taken into account at $t > \tau_{in}$, since the electron does not have time to move across the field **H** a distance $\sim d$ in a time τ_{in}.[4–6] The mean square of the displacement along the field **H** in this case is

$$\overline{x^2(t)} = \sum_i \frac{c^2}{H^2}\left\{\int_0^t dt' \int_{-\infty}^{\infty} P(z,t')E_{iy}(z)\,dz\right\}^2 \sim \left(c\frac{E_d}{H}P_2 d\cdot t\right)^2 N_t \ln(d/\rho(t))$$

$$\sim \left(\frac{ce}{\epsilon H}\right)^2 N D_{\parallel}^{-1/2} t^{3/2} \ln(d/\rho(t)), \quad (7)$$

where $P(z,t) \sim (D_{\parallel} t)^{-1/2}\exp(-z^2/2D_{\parallel} t)$, E_{iy} is the field of the i-th impurity, $P_2 \sim (D_{\parallel} t)^{-1/2}$, $N_t \sim Nd^2(D_{\parallel} t)^{1/2}$ is the number of impurities with which the electron interacts in a time t, and $\rho(t) = [x^2(t) + y^2(t)]^{1/2}$; the summation is over all the impurities. We see from (7) that for $x(t) < d$ the motion is nondiffusive in nature. The time required for the electron to move across the field **H** a distance d, the third scale time of interest to us, can be found from the condition $\overline{x^2(\tau_3)} \sim d^2$, $\ln[d/\rho(\tau_3)] \sim 1$.

4. At $t > \tau_3$ the transverse motion becomes diffusive. The diffusion coefficient is

$$D_{\perp} = \frac{d^2}{\tau_3} = \alpha \left(\frac{ec}{\epsilon H}\right)^{4/3} N^{2/3} d^{2/3} D_{\parallel}^{-1/3}, \tag{8}$$

where $\alpha \sim 1$ is a numerical coefficient. The conductivity of the degenerate electron gas across the field **H** is

$$\sigma_{\perp} = \alpha \left(\frac{e^3 c \partial n / \partial \mu}{\epsilon H} \right)^{4/3} N^{2/3} d^{2/3} \sigma_{\parallel}^{-1/3}, \tag{9}$$

where $\partial n/\partial \mu$ is the state density at the Fermi level. In Eqs. (8) and (9) the localization effects are taken into account in terms of $D_{\parallel}$ and $\sigma_{\parallel}$. It follows from Eqs. (6) and (9) that $\sigma_{\parallel}$ decreases and $\sigma_{\perp}$ increases with decreasing temperature.

5. With increasing magnetic field, k_z^{-1} becomes larger than the Debye screening length $[(4\pi e^2/\epsilon)\partial n/\partial \mu]^{-1/2}$ and the impurity potential becomes strongly anisotropic.[8,9] Our analysis is correct even if d in (8) and (9) is taken to mean the screening length across the field **H**.

6. At $T > 2$ K, $\sigma_{\parallel}$ was found to decrease in n-InAs and $\sigma_{\perp}$ was found to increase with decreasing temperature in the ultraquantum limit.[10] In the n-InSb sample the $\sigma_{\perp}$

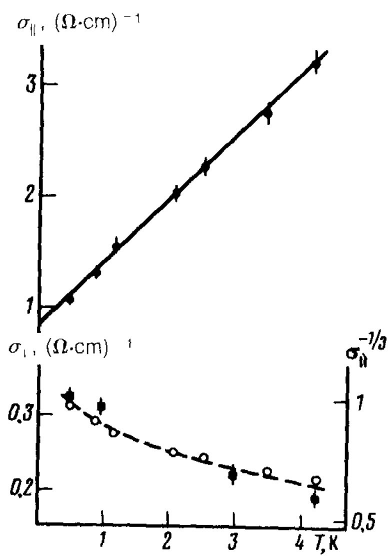

FIG. 1. The temperature dependence of $\sigma_{\parallel}$ (♦), $\sigma_{\perp}$ (♦), and $\sigma_{\parallel}^{-1/3}$ (O) in a field $H = 10$ kOe. The electron density of the sample n-$Hg_{0.79}Cd_{0.21}Te$ is $n = 2.7 \times 10^{14}$ cm^{-3}. The plots are based on the data of Ref. 12. The origin of the scale for $\sigma_{\perp}$ is the same as that for $\sigma_{\parallel}^{-1/3}$.

was also found to increase with decreasing temperature.[11] In the region of interest the most comprehensive data, which allowed us to construct the temperature dependences of $\sigma_\perp$ and $\sigma_\parallel$, were obtained in Ref. 12 with use of a n-$Hg_{0.79}Cd_{0.21}Te$ sample with the electron density $n = 2.7\times 10^{14}$ cm^{-3}. The curves are plotted in Fig. 1. It is possible for $\sigma_\parallel$ to depend linearly on T if τ_{in} is determined by the scattering by phonons with the wave vector $q_T \sim kT/\hbar s \gg \lambda^{-1}$. The transverse conductivity increases with decreasing temperature. As can be seen in Fig. 1, the T dependence of $\sigma_\perp$ is similar to the T dependence of $\sigma_\parallel^{-1/3}$; i.e., the condition $\sigma_\perp \propto \sigma^{-1/3}$ is satisfied. The Hall conductivity, which is virtually independent of the temperature, is $\sigma_H = 0.5$ S/cm.

I wish to thank V. F. Gantmakher, S. V. Meshkov, M. R. Trunin, and D. E. Khmel'nitskiĭ for useful discussions.

[1)]More exactly, instead of (2) we can write $(\overline{v^2})^{1/2} \sim (c/H)\{\int_{\rho(t)}^{d}[(e/\epsilon\rho'^2)\times(\rho'/l)]^2 Nl\, 2\pi\rho' d\rho'\}^{1/2} \sim (ecN^{1/2}/\epsilon H l^{1/2})\ln[d/\rho(t)]$, where ρ' is the distance between the impurity and the trajectory of the electron along the field **H**, and $\rho(t)$ is the displacement of the electron across the field **H** in a time t.

[1]F. Adams and T. Holstein, J. Phys. Chem. Solids **10**, 254 (1959).
[2]R. Kubo, S. M. Mijake, and N. Nashitsume, Solid State Physics **17**, 269 (1969), New York and London, Academic Press.
[3]A. A. Abrikosov and I. A. Ryzhkin, Adv. Phys. **27**, 147 (1978).
[4]Yu. A. Dreizin and A. M. Dykhne, Sixth European Conf. on Controlled Fusion and Plasma Physics, Moscow, **1**, 147, 1973.
[5]S. S. Murzin, Pis'ma Zh. Eksp. Teor. Fiz. **39**, 567 (1984) [JETP Lett. **39**, 695 (1984)].
[6]D. G. Polyakov, Zh. Eksp. Teor. Fiz. **90**, 546 (1986) [Sov. Phys. JETP **63**, 317 (1986)].
[7]V. L. Berezinskiĭ, Zh. Eksp. Teor. Fiz. **65**, 1251 (1973) [Sov. Phys. JETP **38**, 620 (1974)].
[8]N. J. Horing, Ann. Phys. **54**, 405 (1969).
[9]B. I. Shklovskiĭ and A. L. Éfros, Zh. Eksp. Teor. Fiz. **64**, 2222 (1973) [Sov. Phys. JETP **37**, 1122 (1973)].
[10]S. S. Murzin, Pis'ma Zh. Eksp. Teor. Fiz. **44**, 45 (1986) [JETP Lett. **44**, 56 (1986)].
[11]S. Ishida and E. Otsuka, J. Phys. Soc. Jpn. **43**, 124 (1977).
[12]M. Shayegan, V. J. Goldman, H. D. Drew, D. A. Nelson, and P. M. Tedrow, Phys. Rev. B **32**, 6952 (1985).

Translated by S. J. Amoretty