

Mesoscopic fluctuations of thermoelectric coefficients

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The mesoscopic fluctuations of the thermoelectric coefficients are calculated for metallic samples of arbitrary dimensionality. The results show that these fluctuations do not contain the small factor T/μ , which is characteristic of the mean values, and that they can be comparable to or greater than the mean values.

The thermoelectric coefficients which describe the appearance of an electric current under the influence of a temperature gradient β_{ik} and a heat flux in an electric field γ_{ik} are related by the Onsager relation $\gamma_{ik} = -T\beta_{ki}$.

In metals these coefficients have a characteristic small factor T/μ , where μ is the chemical potential. We know that the reason for this circumstance is that the temperature gradient causes particles above the Fermi level and holes below it to move in the same direction, so that the electric currents set up by each of these fluxes separately cancel each other out to within terms on the order of T/μ .

Our purpose in this letter is to show that this cancellation does not occur in mesoscopic metallic systems, and in this sense the mesoscopic fluctuations of the thermal electric coefficients are anomalously large. The reason is that the random scattering potential completely eliminates the symmetry between the electrons and the holes,

and the fluxes of particles in holes cancel out only on the average. In other words, the mean value of the conductivity changes substantially when the energy of the particles changes by an amount on the order of the Fermi energy. Mesoscopic fluctuations of the conductivity vary as a function of the energy over distances on the order of $\max\{E_c, T\}$, where $E_c = D/L^2$ is the reciprocal of the time required for an electron to cross a sample of size L by diffusion. According to the Mott formula, which also holds for fluctuations in the case of elastic scattering, the fluctuations of the thermo-emf must therefore be larger by a factor of $\mu/\max\{E_c, T\}$.

The correlation functions of the thermoelectric coefficients for samples of different dimensionalities are

$$\langle \delta\beta_{ik} \delta\beta_{lm} \rangle = \left(\frac{e}{\pi\hbar} \right)^2 \{ (\delta_{il}\delta_{km} + \delta_{im}\delta_{kl}) \Lambda_d + \delta_{ik}\delta_{lm} \Xi_d \}, \quad (1)$$

where

$$\Lambda_d = \frac{V_d \cdot L T^{4-d}}{V_3^2} \left\{ \begin{array}{ll} I_3; & d = 3 \\ 2I_2 \ln \frac{L_{in}}{L_T}; & d = 2, \\ 2\pi I_2 \frac{L_{in}}{L_T}; & d = 1 \end{array} \right. \quad (2)$$

$$\Xi_d = \frac{V_d L T^{4-d}}{V_3} I_d 2^{3-d} \quad (3)$$

Here $L_T = \sqrt{D\hbar/T}$, $L_{in} = \sqrt{D\tau_{in}}$, τ_{in} is the relaxation time with respect to inelastic scattering processes, D is the electron diffusion coefficient, and V_d is the d -dimensional volume ($V_1 = L$ is the length of a wire; V_2 is the area of a film; and V_3 is the volume of a sample). The effective dimensionality of the sample in expressions (2) and (3) is found with respect to the length L_T . Finally,

$$I_d = \frac{2^{d/2}}{6} \left\{ \frac{d}{2} \left(1 + \frac{d}{2}\right) \Gamma\left(2 - \frac{d}{2}\right) \zeta\left(1 - \frac{d}{2}\right) + \frac{1}{2} \Gamma\left(4 - \frac{d}{2}\right) \zeta\left(3 - \frac{d}{2}\right) \right\},$$

where $\zeta(x)$ is the Riemann zeta function.

If we introduce a thermoelectric conductance in accordance with

$$J = \frac{1}{e} B (T_1 - T_2),$$

we find the following expressions for the correlation function $\langle \delta^2 B \rangle$ in the different dimensionalities:

$$\langle \delta^2 B \rangle = \left(\frac{e^2}{\pi\hbar} \right)^2 \frac{V_d L T^{4-d}}{L^4} \left\{ \begin{array}{ll} \frac{5}{2} I_3; & d = 3 \\ 4I_2 \ln \frac{L_{in}}{L_T}; & d = 2. \\ 4\pi I_2 \frac{L_{in}}{L_T}; & d = 1 \end{array} \right. \quad (4)$$

This concept of a thermoelectric conductance is convenient in that it is also meaningful in the zero-dimensional case, in which it is necessary to introduce a local temperature gradient. In such samples, an equilibrium distribution of electrons with the given temperature is established only over a distance $\sqrt{D\tau_e} > L$, where τ_e is the energy relaxation time of the electrons. In this situation, however, it is possible to find the heat flux which is set up by a uniform electric field, and it is possible to find the thermoelectric conductance of the sample from the Onsager relation. The thermoelectric conductance determined in this way for the zero-dimensional case is

$$\langle \delta B^2 \rangle = 0,1 \left(\frac{e^2}{\pi \hbar} \frac{V_1^2}{L_T^2} \right)^2 \propto T^2; \quad L_T > L. \quad (4a)$$

As in the conductance fluctuations in Ref. 2, the first term in (1) describes the part of the thermoelectric coefficient which stems from correlations in the diffusion coefficient, while the second describes the part due to correlations in the state density.

Let us compare the magnitude of the fluctuations in the thermoelectric coefficient with its mean value $\langle \beta \rangle = T\sigma/e\mu$. In the zero-dimensional case we have

$$\frac{\langle \delta^2 \beta \rangle^{1/2}}{\langle \beta \rangle} \simeq \frac{e^2}{\hbar} R \frac{\mu}{E_c}.$$

In the other dimensionalities we have

$$\frac{\langle \delta^2 \beta \rangle^{1/2}}{\langle \beta \rangle} \simeq \frac{e^2}{\hbar} \frac{\mu}{T} \begin{cases} R \left(\frac{L_T V_3}{L^4} \right)^{1/2}; & d = 3 \\ R_{\square} \left(\frac{L_T^2}{V_2} \ln \frac{L_{in}}{L_T} \right)^{1/2}; & d = 2. \\ R \left(\frac{L_T^2 L_{in}}{L^3} \right)^{1/2}; & d = 1 \end{cases}$$

Here R is the resistance of the sample, and R_{\square} is the surface resistivity of a film. It can be seen from these estimates that the fluctuations in the thermoelectric coefficients in fact do not contain the small factor T/μ which is characteristic of the mean thermoelectric coefficients. These fluctuations may be comparable to or even greater than the mean values. The absolute value of the fluctuations in the thermoelectric conductance, on the other hand, are on the order of the fluctuations of the ordinary electrical conductance G at $T > E_c$.

The correlation function for the fluctuations in the thermoelectric currents at a given point with a uniform temperature gradient is, in three dimensions,

$$\langle \delta j^2(\mathbf{r}) \rangle \simeq 8 \left(\frac{L_T}{l} \right)^5 (\langle \bar{\beta} \rangle \nabla T)^2.$$

The local thermoelectric currents are thus significantly greater than the mean current.

The correlation function for the fluctuations in the ohmic current J and in the heat flux q is $\langle \delta J \delta q \rangle = 0$ in an electric field; i.e., we have $(\langle \delta G \delta B \rangle) / (\langle \delta G^2 \rangle) \approx 0 (T/\mu) \ll 1$. The reason for this result is that the asymmetry between the electrons and the holes is a random function, so that the correlation between δG and δB is zero within terms of order T/μ .

To calculate the correlation functions for the thermoelectric coefficients, it is convenient to calculate the correlation functions of the heat fluxes under the influence of an electric field E . In the thermal vertex we have the quantity

$$\frac{1}{2m} \left[\left(\epsilon - \frac{\Omega}{2} \right) \left(p - \frac{k}{2} \right)_i + \left(\epsilon + \frac{\Omega}{2} \right) \left(p + \frac{k}{2} \right)_i \right]$$

here. The important diagrams are the same as the corresponding important diagrams for calculating the fluctuations of the total conductance.^{1,2} All diagrams in which the sign of the energy does not change at the vertex are identically zero, as they are in calculation of the conductance.

As usual in mesoscopic experiments,^{1,4,5} the particular realization of a random potential can be changed by changing the magnetic field. In this case the local fluctuations of the thermoelectric current lead, by virtue of the continuity equation, to spatial fluctuations of the electric potential $\langle \delta^2 \varphi(r) \rangle \approx (L_T/l) (\nabla T \hbar / e p_F)^2$. This estimate agrees with the estimate of fluctuations in a uniform electric field,³ $\langle \delta^2 \varphi(r) \rangle$, when E is replaced by $(1/e) \nabla T$. In experiments of that sort, however, it is considerably simpler to produce a uniform temperature gradient than a uniform gradient of the electric field, since the temperature distribution is determined by not only the thermal conductivity of the sample but also the substrate.

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