Retention time of atoms and their density modulation in a resonant field of a standing wave

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The lifetime of atoms trapped at the antinodes of a standing-wave field is found. The steady-state amplitude of spatial modulation of the distribution function with a half-wave period is found.

The control of motion of neutral atoms by means of resonant light pressure has recently been studied extensively, both experimentally and theoretically.[1] An experimental observation of atoms trapped in a confinement system created by a strongly focused laser beam was reported in Ref. 2. Another interesting suggestion for capturing cold atoms at the antinode of a standing light wave (at the potential-energy minimum) was made in Ref. 3. However, since the lifetime of an atom in such a potential well has not been found, it is unclear whether atoms can be held at the potential-energy minima of the field of a standing wave. In this study we found this time, which turns out to be rather short ($\sim 10^{-5}$ s) for typical parameter values. We have also found the function for the steady-state distribution of the atoms, which is modulated with a half-wave period of the wave.

1. The Fokker-Planck equation. We consider the motion of a gas of atoms in the field of a standing wave,

$$E(x, t) = 2E \cos kx \cos \omega t .$$

The atom has the transition frequency ω_{21}. The range of deviation of the field frequency with respect to the transition frequency is assumed to be negative, i.e., $\omega - \omega_{21} = -\Delta$, where $\Delta > 0$. The lower level 1 is the ground level and the upper level 2 decays directly to the ground state with the probability γ. We consider the case in which the transition becomes saturated only slightly, where $\kappa = V^2/[\Delta^2 + (\gamma/2)^2] \ll 1$, $V = dE/\hbar$, and d is the dipole moment of the transition. The ratio R/γ is assumed to be small: $R/\gamma \ll 1$, where $R = \hbar k^2/2M$ is the recoil energy of the atom upon absorption of a photon, and M is the mass of the atom. The velocity v of the atom satisfies the condition $kv \ll \gamma$, i.e., the atoms are assumed to be slow. We have, however, $v \gg u$, where $u = \hbar k/M$ is the recoil velocity.

In the one-dimensional case, where the distribution function for the atoms, $f = f(x,v,t)$, does not depend on y and z, we can use the Fokker-Planck equation[4]

$$\frac{\partial f}{\partial t} + v\frac{\partial f}{\partial x} + \frac{1}{M}\frac{\partial}{\partial v}(F + F_{tr})f = \frac{\partial^2}{\partial v^2}Df, \qquad (1)$$

where

$$F/M = -a, \qquad a = u\Delta\kappa \sin 2kx,$$

$$F_{tr}/M = -qv, \qquad q = 4R\gamma\kappa\frac{\Delta}{\Delta^2 + (\gamma/2)^2}\sin^2 kx,$$

$$D = u^2\gamma\kappa\left(\frac{1}{5}\cos^2 kx + \frac{1}{2}\sin^2 kx\right).$$

Here F is the gradient force which can be expressed in terms of the potential energy as $F = -\partial u/\partial x$, where $u = -u_0\cos^2 kx$, $u_0 = \hbar\Delta\kappa$, u_0 is the potential barrier, F_{tr} is the frictional force, and D is the diffusion coefficient.

2. The lifetime of an atom in the potential well. The potential-energy minima U, which are situated at the antinodes of the field, are spaced a half wavelength $(\lambda/2)$ apart. Let us find a solution of Eq. (1) near the minimum $x = 0$. Assuming $kx \ll 1$, we find the following parameters within the required accuracy:

$$a = \Omega^2 x, \qquad q = 0, \qquad D = D_0 + \Omega^2\beta x^2;$$

$$\Omega^2 = 4R\Delta\kappa, \quad D_0 = \frac{1}{5}u^2\gamma\kappa, \quad \beta = \frac{3}{10}\frac{R\gamma}{\Delta}.$$

Equation (1) can now be written

$$\frac{\partial f}{\partial t} + v\frac{\partial f}{\partial x} = \Omega^2\frac{\partial f}{\partial v} + (D_0 + \Omega^2\beta x^2)\frac{\partial^2 f}{\partial v^2} .$$

We find the Fokker-Planck equation for an oscillator with a diffusion coefficient which depends quadratically on x. From this equation for the average kinetic energy of the atoms

$$\epsilon = \int_{-\infty}^{\infty} dv \int_{-\infty}^{\infty} dx \frac{Mv^2}{2} f(x, v, t)$$

we find

$$\mathcal{E} = \mathcal{E}(0) e^{\beta t} + \frac{2}{3} U_0 (e^{\beta t} - 1).$$

We see that in a time

$$\tau = 1/\beta = \frac{10}{3} \frac{\Delta}{R\gamma} \qquad (2)$$

the atom is "forced out" of the potential well as a result of diffusion.

3. The steady-state solution. In the steady state we must solve the equation

$$v \frac{\partial f}{\partial x} - \frac{\partial}{\partial v} (a + qv) f = \frac{\partial^2}{\partial v^2} D f.$$

We recall that a, q, and D are functions of x. We will seek a solution of this equation on the basis of the perturbation theory, making use of the small value of the parameter κ:

$$f(x, v) = f_0(v) + f_1(x, v),$$

where f_1 is much smaller than f_0 in the parameter κ. We thus see that the distribution function is a Maxwellian function, $f_0(v) \sim \exp(-v^2/v_0^2)$, and that the average energy of the atoms is[4]

$$\mathcal{E} = \frac{Mv_0^2}{2} = \frac{7\hbar}{20} \frac{\Delta^2 + (\gamma/2)^2}{\Delta}. \qquad (3)$$

Knowing f_0, we find f_1 and then the atomic density as a function of x

$$n(x) = \int_{-\infty}^{\infty} dv f(x, v) \sim 1 + \delta n \cos 2kx,$$

where the density modulation amplitude δn is

$$\delta n = \kappa \frac{10}{7} \frac{\Delta^2}{\Delta^2 + (\gamma/2)^2}. \qquad (4)$$

4. Discussion. Let us now consider the physical behavior of the atomic gas in the field of a standing wave. In this steady state an equilibrium is established between the frictional force, which cools the atoms, and the atomic diffusion (heating), which increases the energy of the atom in this case. Since the atom is forced out of the potential well because of the diffusion, we will have uncaptured atoms in the steady state, whose energy is above the potential barrier U_0. We thus have two ensembles of atoms: atoms with energy above the potential barrier U_0 and atoms which are trapped in the potential wells of the field of a standing wave. According to (3) and (4), the number of atoms in the potential wells is $\delta n \sim U_0/\mathscr{E} \sim \kappa \ll 1$. There is an exchange of atoms between these ensembles in a time which is in order of magnitude $\tau \sim \Delta/R\gamma$. Qualitatively, our analysis is valid even for $\kappa \sim 1$. At $\Delta \sim \gamma$ we have $\tau \sim 1/R$. Although at $\Delta/\gamma \gg 1$ the time increases, we must increase the field strength appreciably in order to satisfy the condition $\kappa \lesssim 1$. Otherwise, the potential wells will have a negligible number of particles.

The transition of the sodium atom, $3S$–$3P$ ($\lambda = 5890$ Å, $3S$ is the ground state) is generally used in the experiments, $R = 1.5 \times 10^5\ s^{-1}$, $R/\gamma = 5 \times 10^{-3}$, the saturation parameter is $\kappa = I/I_s\{(\gamma/2)^2/[\Delta^2 + (\gamma/2)^2]\}$, where I is the intensity of the standing wave, and $I_s \sim 0.1\ W/cm^2$. The lifetime is $\tau \sim \Delta/\gamma \times 10^{-5}$ s, i.e., at $\Delta \sim \gamma$ it is on the order of 10^{-5} s.

5. *Conclusions.* In the antinodes of the resonant light field of a standing wave the atoms have a rather short lifetime ($\sim 10^{-5}$ s for typical parameter values: $\kappa \sim 1$, $\Delta \sim \gamma$), i.e., they are actually not trapped. In the case of large frequency deviations, this time can be increased, but the field strength in this case will increase appreciably (at $\tau \sim 0.1$ s it is necessary that $\Delta/\gamma \sim 10^4$ for $I \sim 10^7\ W/cm^2$). The periodicity of the potential leads to a modulation of the atomic density with a modulation amplitude of about κ.

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