Solitons in Josephson superlattices

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(Submitted 30 January 1987)
Pis'ma Zh. Eksp. Teor. Fiz. **45**, No. 6, 299–301 (25 March 1987)

Equations describing the behavior of solitons (magnetic vortices) in multilayer Josephson systems are derived. If the velocities of solitons propagating in two adjacent junctions differ only slightly, they will capture each other when they come close together and thereafter propagate at the same velocity.

Recent years have seen the development and study of Josephson superlattices, i.e., systems consisting of alternating layers *S–N–S–N–*... or *S–I–S–I–*..., where *S* is a superconductor, *N* a normal metal, and *I* an insulator.[1] From another standpoint, a long *S–I–S* Josephson junction ($L \gg \lambda_J$, where L is the length of the junction, and λ_J is the Josephson penetration depth) is a convenient system for studying solitons or "fluxons," as they are called in this case. At certain currents and voltages V, moving fluxons and/or antifluxons form in such junctions. From the $I(V)$ curve one can draw definite conclusions regarding the dynamics and interaction of fluxons (or antifluxons).[2]

The interaction of fluxons in multilayer systems is an interesting topic. Most of the previous theoretical work on this topic has dealt with the emission from multilayer systems.[3,4] At the same time, such systems open up some new opportunities for studying the interaction of fluxons, since by varying the parameters of the system and the external conditions (the magnetic field and the currents through junctions) one can control the characteristics of the fluxons in different junctions (their velocities, the period of the vortex lattice, etc.) In the present letter we derive equations describing the behavior of solitons in multilayer systems. We analyze several effects in such systems.

Let us examine an S_1–I–S_2–I–... system (Fig. 1). Assuming a local coupling of the current $\mathbf{j}_\perp$ in the plane of the n-th S layer with a momentum $\mathbf{p}_S$,

$$\mathbf{j}_\perp = b\mathbf{p}_S, \qquad \mathbf{p}_S = \vec{\nabla}_\perp \chi - (2e/c)\,\mathbf{A}\,, \tag{1}$$

we find the following expression for the magnetic field $\mathbf{H}(z)$ in the n-th S layer from the London equation:

$$\mathbf{H}_n(z) = \frac{1}{2}\left[(\mathbf{h}_n - \mathbf{h}_{n-1})\,\frac{\sinh(z/\lambda)}{\sinh(a_n/\lambda)} + (\mathbf{h}_n + \mathbf{h}_{n-1})\,\frac{\cosh(z/\lambda)}{\cosh(a_n/\lambda)}\right], \tag{2}$$

where $b^{-1} = 8\pi e\lambda^2/c^2$, λ is the London penetration depth, and h_n is the magnetic field in the n-th I layer (Fig. 1). Making use of the continuity of A, we can write the equation $\mathbf{j}_\perp(a_n + 0) - \mathbf{j}_\perp(a_n - 0) = b\vec{\nabla}_\perp \varphi_n$, for the jump in the current at this layer, where φ_n is the phase difference at this I layer. Using Maxwell's equation

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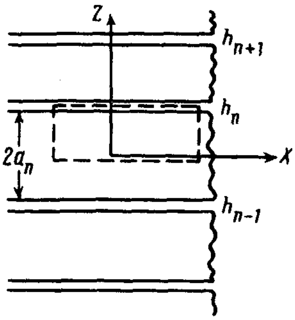

FIG. 1.

$$\frac{4\pi}{c}\,\mathbf{j}+\frac{\epsilon}{c}\,\frac{\partial E}{\partial t}=\operatorname{curl}\mathbf{H} \tag{3}$$

we can express $\mathbf{j}_\perp(a_n\pm 0)$ in terms of the fields $\mathbf{H}_n$ and $\mathbf{H}_{n+1}$, ignoring the fields $\mathbf{E}_\perp$ in the S layers. We find

$$\begin{aligned}-(c/e\lambda)\,[\mathbf{n}_z\times\vec{\nabla}_\perp\varphi_n]\\ =(\mathbf{h}_{n+1}-\mathbf{h}_n)\coth\tilde{a}_{n+1}-(\mathbf{h}_{n+1}+\mathbf{h}_n)\tanh\tilde{a}_{n+1}-(\mathbf{h}_n-\mathbf{h}_{n-1})\coth a_n\\ -(\mathbf{h}_n+\mathbf{h}_{n-1})\tanh\tilde{a}_n,\end{aligned} \tag{4}$$

where $\tilde{a}_n=a_n/\lambda$. We now take the divergence of Eq. (3) and integrate it over the volume shown by the dashed box in Fig. 1. Using the relationship between $\mathbf{j}_\perp$ and $\mathbf{H}$ described by Eq. (3), and using expression (2) for $\mathbf{H}$, we find the equation

$$j_{c,\,n}\sin\varphi_n+(\sigma_n/2e)\dot{\varphi}_n-(c/4\pi)\,[\mathbf{n}_z\times\vec{\nabla}_\perp]\,\mathbf{h}_n+(\epsilon/8e\pi d_n)\ddot{\varphi}_n=0. \tag{5}$$

The first two terms here describe the Josephson and dissipative currents through the n-th I layer, of thickness d_n; the last term describes the displacement current; and $\mathbf{n}_z$ is the unit vector along the z axis. Equations (4) and (5) describe the electrodynamics of this system. Let us examine some specific examples.

a) System of identical junctions with thin ($a_n\equiv a\ll\lambda$) S layers (a layered superconductor). From (4) we find for $\mathbf{h}=(0,h,0)$ the relation $-(2\pi b/ac)\varphi_x=h_{zz}$ in this case (where $h_{zz}=\partial^2h/\partial z^2$). In the steady state, this equation, along with (5), describes a vortex (or a vortex lattice). Far from the center of an isolated vortex, where $\sin\varphi\approx\varphi$, we find $\kappa^2h_{zz}+h_{xx}=0$ from these equations (here $\kappa^2=16\pi eaj_c\lambda^2/c^2\equiv a\lambda/\lambda_J^2\ll 1$). This equation describes an anisotropic vortex in a layered superconductor.[5]

b) System of junctions with thick ($a_n\gg\lambda$) S layers. From (4) we find

$$h_n=(2\pi\lambda b/c)\,[\varphi_n'+\gamma_{n+1}\varphi_{n+1}'+\gamma_{n-1}\varphi_{n-1}'], \tag{6}$$

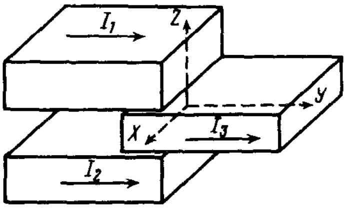

FIG. 2.

where $\gamma_n = \exp(-2a_n/\lambda) \ll 1$, and the prime means differentiation with respect to x. Substituting (6) into (5), we find, in the steady state,

$$\sin\varphi_n - \lambda_{J,n}^2 [\varphi_n'' + \gamma_{n+1}\varphi_{n+1}'' + \gamma_{n-1}\varphi_{n-1}''] = 0. \tag{7}$$

This equation describes a vortex lattice in weakly interacting Josephson junctions. By varying the coupling between junctions (by varying the temperature or the thickness of the S layers) and the structure of the system (which may be periodic or aperiodic), one can produce various types of vortex lattices and thus change the dependence of the magnetization on the external magnetic field. Leaving this question for future study, we move on to the next case.

c) System comprised of two junctions (Fig. 2). In the case of a single junction, this geometry is used to study the motion of solitons.[2] It is assumed that the dimension along the y axis is small in comparison with λ_J. In this case h and φ in (5) depend only weakly on y, and we can integrate (5) over y from $-L_y/2$ to $L_y/2$. We express the field components $h_x(\pm L_y/2)$ which arise in this process in terms of the currents I, integrating (3) over the area of each S layer. Assuming that the coupling between junctions is weak, as in the preceding case, we find the equation

$$\sin\varphi_{1(2)} - \lambda_{J,1(2)}^2 [\varphi_{1(2)}'' + \gamma\varphi_{2(1)}''] + (\alpha/\omega)_{1(2)}\dot{\varphi}_{1(2)} + \omega_{1(2)}^{-2}\ddot{\varphi}_{1(2)} = \eta_{1(2)}, \tag{8}$$

where $(\alpha/\omega)_{1(2)} = (\sigma/j_c)_{1(2)}/2e$, $\omega_{1(2)}^{-2} = 8e\pi(j_c d/\epsilon)_{1(2)}$, and $\eta_{1(2)} = \mp(I/j_c)_{1(2)}(L_x L_y)^{-1}$. If the system is in a magnetic field $(0,h,0)$, the boundary condition on (8) is $\varphi'_{1(2)} = (4e\lambda/c)h$.

For simplicity, we consider two identical junctions with different "currents" η_1 and η_2, at which fluxons propagate in the junctions. If $\eta_1\eta_2 > 0$ (i.e., if the currents I_1 and I_2 are flowing in the same direction), the fluxons in junctions 1 and 2 will move in different directions, and we can study a collision between them. If, on the other hand, the condition $\eta_1\eta_2 < 0$ holds (I_1 and I_2 are flowing in different directions), the fluxons will move in the same direction, at velocities v_1 and v_2 determined by the currents η_1 and η_2. It turns out that, depending on the value of the quantity $\eta_- \equiv \eta_1 - \eta_2$ (we note that we have $\eta_- \sim I_3$), the relative velocity $v_- = v_1 - v_2$ can take on different values. This conclusion follows from the equation for the distance between fluxons, $x_- \equiv \xi\lambda_J$, which we write under the assumption that $v_{1(2)}$ is small in comparison with $c_0 = \omega_0\lambda_J$ (where $\omega_0 = \omega_1 = \omega_2$) when the width of a fluxon does not depend on its velocity:

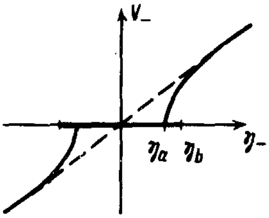

FIG. 3.

$$\omega_0^{-2}\ddot{\xi} + (\alpha/\omega_0)\dot{\xi} + (\pi/4)\eta_- + 2\gamma F(\xi) = 0, \qquad (9)$$

where $F(\xi) = \int_{-\infty}^{\infty} dx f'(x) f''(x+\xi)$ and $f(x) = 4\arctan(\exp x)$. It follows from (9) that in the case $\eta_- \geqslant \eta_a \equiv 8a\sqrt{\gamma}/\pi$ the velocity is $v_- = \dot{x}_- = 0$; i.e., the fluxons are moving as a bundle. In the case $\eta_- > \eta_b \equiv 0.79\gamma$, the velocity v_- is nonzero, and in a long junction it is equal to $v_- = -\eta_-\omega_0\pi/4\alpha$ most of the time. In the current interval $\eta_a < \eta < \eta_b$, two types of motion are possible. The voltages across the junctions, V, are related to the average velocity of the motion: $2eV_{1(2)} = \pi\bar{v}_{1(2)}/L$. Consequently, we can draw conclusions from these results regarding, for example, the dependence of $V_- \equiv V_1 - V_2 = \pi\bar{v}_-/eL$ on η_-. This dependence is sketched in Fig. 3. In the limit $\eta_- \to \eta_a$ we have $V_- \sim \sqrt{\gamma}\ln\sqrt{\gamma/\eta_a(\eta_- - \eta_a)}$; as η_- increases, the $V_-(\eta_-)$ dependence rapidly becomes the linear dependence $eV_- = \pi|\eta_-|\omega_0\lambda_J|4e\alpha L$. In the equations written above we have assumed $\gamma < \alpha \ll \sqrt{\gamma}$.

There are other effects that occur in these systems. For example, if we place a system (Fig. 2) with different junctions in a magnetic field, a vortex lattice may arise in one of the junctions, and a fluxon in another junction will move in the periodic potential set up by the vortex lattice.[6]

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Translated by Dave Parsons