Production and capture of neutrons in an electric field

O. A. Pankratov
P. N. Lebedev Physics Institute, Academy of Sciences of the USSR

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Bound states of a neutron, which are due to the interaction of its magnetic moment with the electric field, are shown to exist near a flat charged plate. A finite baryon vacuum charge arises simultaneously. The sign of this charge changes when the electric charge of the plate changes its sign.

The interaction of a neutron with the external electromagnetic field is described by the Dirac equation

$$(i\hbar c\ \gamma^\mu \partial_\mu + \mu_a \frac{i}{2}\gamma^\mu \gamma^\nu F_{\mu\nu} - Mc^2)\,\psi = 0, \tag{1}$$

where $\mu_a = -1.91\ \mu_n$ (μ_n is the Bohr nuclear magneton). For a static electric field **E** Eq. (1) reduces to

$$\begin{pmatrix} Mc^2 - \epsilon & c\vec{\sigma}\hat{\mathbf{p}} - i\mu_a \vec{\sigma}\mathbf{E} \\ c\vec{\sigma}\hat{\mathbf{p}} + i\mu_a \vec{\sigma}\mathbf{E} & -Mc^2 - \epsilon \end{pmatrix} \begin{pmatrix} \psi_1 \\ \psi_2 \end{pmatrix} = 0. \tag{2}$$

Let us consider the one-dimensional case $\mathbf{E}(\mathbf{r}) = [0,0,E_z(z)]$. We assume that the asymptotic cases $E_z(z\to \pm\infty) = \pm E_\infty$ have opposite signs. A uniformly charged flat plate, for example, produces such a field. The problem involving the motion of a neutron in such a field is the same as the problem involving a counter-propagating ferroelectric domain wall in a semiconductor with Dirac bands, which was considered elsewhere.[1] According to Volkov and Pankratov,[1] regardless of the particular form of $E_z(z)$, there are fermionic states localized along z, whose energy is independent of the momentum $P_\perp$ in the plane perpendicular to the z axis:

$$\epsilon_\pm = \pm Mc^2. \tag{3}$$

Here the plus sign in (3) is used when the charge of the plate is positive and the minus sign is used when it is negative.

Hamiltonian (2) can be diagonalized with respect to spin variable by means of a simple unitary transformation. As a result, we obtain the equations

$$\begin{pmatrix} Mc^2 - \epsilon & ic\hat{p}_z + \mu_a E_z(z) \pm cp_\perp \\ -ic\hat{p}_z + \mu_a E_z(z) \pm cp_\perp & -Mc^2 - \epsilon \end{pmatrix} \begin{pmatrix} \phi_1^\pm \\ \phi_2^\pm \end{pmatrix} = 0, \tag{4}$$

which, taken collectively, are unitarily equivalent to (2). Such a transformation brings about a transition from the functions $\psi_{1,2}$ to the spinors $\phi^{\pm}_{1,2}$, which are the eigenfunctions of the operator $\mathbf{n}_z\,[\sigma\times\mathbf{p}_\perp]$ ($\mathbf{n}_z$ is a unit vector along the z axis). As a result, the spin structure of the wave function with a given momentum $\mathbf{p}_\perp$ is fixed in a similar way the states of a massless particle are automatically spiral states.

We assume that in Eq. (4), for example, $\epsilon = Mc^2$. We will then have $\phi^{\pm}_2 = 0$ and $\phi^{\pm}_1$ will satisfy the equation

$$(-ic\hat{p}_z + \mu_a E_z(z) \pm cp_\perp)\,\phi^{\pm}_1 = 0. \qquad (5)$$

The solutions of this equation

$$\phi^{\pm}_1(z) = \phi_1(0)\exp\left\{-\frac{1}{\hbar c}\int_0^z [\mu_a E_z(z) \pm cp_\perp]\,dz\right\} \qquad (6)$$

are normalizable if $cp_\perp < |\mu_a E_\infty|$. Consequently, the number of localized states (per unit area) is

$$N_0 = 2\,\frac{\pi (p_\perp^{max})^2}{(2\pi\hbar)^2} = \frac{\mu_a^2 E_\infty^2}{2\pi\hbar^2 c^2}\,. \qquad (7)$$

The 2 in Eq. (7) takes into account the double degeneracy of each state with a given momentum $\mathbf{p}_\perp$ which corresponds to two functions in (6). Equation (7) can be rewritten in the form $N_0 = 1/2\pi l_0^2$, where $l_0 = \hbar c/|\mu_a E_\infty|$ is the localization length of the functions in (6). Estimate for the field $E_\infty \approx 10^9$ V/cm gives $|\mu_a E_\infty| \approx 2\times 10^{-5}$ eV and $l_0 \approx 1$ cm. The quantity $\mu_a^2 E_\infty^2/2Mc^2$ is the binding energy. In a static field $E_z(z) \equiv E =$ const, from (4) we find the spectrum of the delocalized states

$$\epsilon_\pm(p_z, \mathbf{p}_\perp) = \pm\,[M^2c^4 + c^2p_z^2 + (cp_\perp \pm \mu_a E)^2]^{1/2}\,. \qquad (8)$$

We see from Eq. (8) that the energy gap between the levels (3) and spectrum (8) with $p_z = p_\perp = 0$ is $\approx \mu_a^2 E^2/2Mc^2$.

The states $\epsilon_\pm$ are the null modes of the Dirac Hamiltonian. As is well known, these states lead to a nonzero vacuum charge by means of the fermion-splitting mechanism,[2] because the null-mode states are formed from both Dirac bands.[1)] The number of levels with a negative energy in this case does not equal to the number of states with a positive energy. In our case, for example, when the charge of the plate is negative, the number of states in the null mode, $\epsilon_- = -Mc^2$, does not equal to the decrease in the number of states in the Dirac band (8) with a negative energy. Some of the null-mode states (the levels which have come from a band with a positive energy) are therefore free states, consistent with the appearance of antineutrons!

The extent to which the conservation of baryon number is violated can easily be determined by noting that problem (4) is equivalent to the theory[3,4] in $(1+1)$ dimensions. The only difference from the case considered by Brazovskiĭ *et al.*[4] is that for $p_\perp \neq 0$ the configuration of the soliton field $W(z) = E_z(z) \pm cp_\perp$ is not antisymmetric, i.e., $|W(-\infty)| \neq |W(+\infty)|$. Clearly, this circumstance leads only to a numerical factor C (according to Goldstone and Wilczek,[3] $1/2 < C < 1$) in the equation for the induced charge.[3,4] As a result, the charge of the Dirac vacuum is

$$\langle N \rangle = -\frac{C}{\pi} N_0 \arctan (\mu_a E_\infty / Mc^2) . \qquad (9)$$

Estimate (9) in the field $E_\infty \approx 10^9$ V/cm gives a catastrophically small value $\langle N \rangle \approx 10^{-14}$ cm^{-2}. Nevertheless, $\langle N \rangle$ is not an exponentially small value in the electric field: $\langle N \rangle \sim E_\infty^3$. Note that $\langle N \rangle$ increases as M^{-4} with decreasing fermion mass.

Eliminating the component $\phi_2^\pm$ from (4), we find

$$[c^2 \hat{p}_z^2 + \hbar c \mu_a E_z'(z) + \mu_a^2 E_z^2 (z) \pm 2cp_\perp \mu_a E_z (z) + c^2 p_\perp^2 + M^2 c^4 - \epsilon^2] \phi_1^\pm = 0. \qquad (10)$$

Because of the presence of a potential step $2cp_\perp \mu_a E_z(z)$, the motion is infinite only in one direction for the energies

$$\epsilon_- (p_z = 0, \mathbf{p}_\perp) < \epsilon < \epsilon_+ (p_z = 0, \mathbf{p}_\perp) \qquad (11)$$

This means that if the momentum p_z of the neutrons situated in the branch $\epsilon_-(p_z, \mathbf{p}_\perp)$ [Eq. (8)] is reduced while holding the energy constant, the neutrons will be reflected completely from the plate at $p_z < (p_\perp |\mu_a E_\infty|/c)^{1/2}$.

The results reported here are strongly linked with the one-dimensional nature of the field **E**. However, such effects can in principle be seen, for example, in strongly deformed nuclei, in particular, in nuclei with high spin.[5,6]

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[1)] The assertion of Volkov and Pankratov[1] that the null mode is split off from either the upper band or the lower band is incorrect.

[1]B. A. Volkov and O. A. Pankratov, Pis'ma Zh. Eksp. Teor. Fiz. **43**, 99 (1986) [JETP Lett. **43**, 130 (1986)].
[2]R. Jackiw and C. Rebbi, Phys. Rev. D **13**, 3398 (1976).
[3]J. Goldstone and F. Wilczek, Phys. Rev. Lett. **47**, 986 (1981).
[4]S. A. Brazovskiĭ, N. N. Kirova, and S. I. Matveenko, Zh. Eksp. Teor. Fiz. **86**, 743 (1984) [Sov. Phys. JETP **59**, 434 (1984)].
[5]A. Zeilinger, C. G. Shull, M. A. Horne, and K. D. Finkelstein, Phys. Rev. Lett. **57**, 3089 (1986).
[6]J. Hamilton, Progr. in Particle and Nuclear Physics **15**, 107 (1985).

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