Oscillations in the transverse magnetoresistance of a point contact

I. B. Levinson, E. V. Sukhorukov, and A. V. Khaetskiĭ
Institute of Problems of the Technology of Microelectronics and Especially Pure Materials, Academy of Sciences of the USSR

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The transverse magnetoresistance of a ballistic point contact is calculated in the model of a "hole in a screen." Classical oscillations of the magnetoresistance are predicted as a function of the magnetic field. These oscillations stem from a geometric resonance between the size of the hole and the Larmor radius.

A point contact is a bridge between two massive parts of a conductor which has dimensions d shorter than the mean free path of an electron, l (a ballistic point contact), or shorter than the electron cooling length $\tilde{l}$ (a diffusion point contact, with $d \gg l$). Research on point contacts was begun by Sharvin,[1] in experiments on the focusing of electrons, and pursued by Jansen *et al.*[2,3] in experiments on the spectroscopy of phonons.

A potential difference V applied to the massive banks falls across a region of size d in the point contact, and it is in this region that the resistance of the point contact, R, is formed. A magnetic field H influences R if the Larmor radius satisfies $r_L \lesssim d$ (for a ballistic point contact) or $r_L \lesssim l$ (for a diffusion point contact). For ordinary point contacts, we would thus have $r_L \lesssim 300$ Å, which corresponds (for copper) to very strong fields, $H \gtrsim 3000$ kOe. A recently developed procedure[4] makes it possible to fabricate ballistic point contacts with dimensions $d \approx 3$ μm. Such point contacts should exhibit a magnetoresistance in attainable fields, $H \approx 30$ kOe.

In the present letter we calculate the resistance of a ballistic point contact, $R(H)$, in the model of a "hole in a screen" for the case in which the field H is parallel to the plane of the screen. At $H = 0$, the resistance of the point contact in this model is[5]

$$R^{-1} = \frac{1}{2} g_F e^2 \int dS \langle v_n(\mathbf{p}) \operatorname{sign}(\mathbf{r}, \mathbf{p}) \rangle . \tag{1}$$

The integration here runs over the points $\mathbf{r}$ of the hole; the angle brackets mean an average over the points $\mathbf{p}$ on the Fermi surface; v_n is the velocity component normal to the plane of the screen; and g_F is the state density on $n\phi$ (the normal is directed along the current). Here $\operatorname{sign}(\mathbf{r},\mathbf{p}) = \pm 1$. Its sign is the same as the sign of the applied potential at the infinity from which the trajectory of an electron which is at point $\mathbf{r}$ with momentum $\mathbf{p}$, arrives. It can be shown that this expression remains valid in the case $H \neq 0$; if we calculate the ohmic resistance R, the trajectories should be constructed without allowance for the electric field near the hole. The calculation of $R(H)$ thus reduces to the problem—simple in principle but exceedingly onerous—of classifying trajectories on the basis of their topology. Below we consider a rectangular hole

$(a \times b)$ with $b \| H$; we assume that the reflection from the plane of the screen is specular; and we assume that $n\phi$ is spherical. Even in this case, the expression for $R(H)$ turns out to be so complicated that we will reproduce here only its asymptotic forms in strong and weak fields H. We introduce a dimensionless field $h = b/2\pi r_L$, where $r_L = v_F/\omega_c$, $\omega_c = eH/mc$ and $\eta = a/b$. If the field is strong, so that we have $r_L \ll a,b (h \gg 1)$, we can write

$$\frac{R^{-1}(H)}{R^{-1}(0)} = \frac{2}{3\pi}\frac{1}{h}\left[\frac{4}{\pi^2} + \frac{1}{\eta} + \frac{6}{\pi^3}\frac{1}{h^{3/2}}\sum_{s=1}^{\infty}\frac{1}{s^{3/2}(4s^2-1)}\sin\left(2\pi s h + \frac{\pi}{4}\right)\right]. \quad (2)$$

In a weak field, with $r_L \gg a,b (h \ll 1)$, we have

$$[R^{-1}(H) - R^{-1}(0)]/R^{-1}(0) = -h^2\varphi(\eta). \quad (3)$$

The explicit expression for $\varphi(\eta)$ is not very important, but we do note that we have

$$\varphi(\eta) = \begin{cases} (\pi^2/3)\eta^2, & \eta \ll 1 \\ (\pi/3)\eta, & \eta \gg 1. \end{cases} \quad (4)$$

How do we interpret these results physically? Let us examine the trajectories of electrons which pass through points of the hole. At $H = 0$, these trajectories are straight lines, while at $H \neq 0$ they consist of sections of spirals (of radius $v_\perp/\omega_c$ with a pitch $2\pi v_\| /\omega_c$), which are connected at the points of reflection from the screen. Here $v_\perp$ and $v_\|$ are the components of the velocity v_F across and along **H**. Figure 1 shows projections of the trajectories onto the plane perpendicular to H.

We first consider the case of strong fields. The trajectories of type 1 intersect the plane of the hole, S, only once, as in the case $H = 0$, but they contribute to the current only on strips of width $v_\perp/\omega_c$ besides the edges of the hole, of length b. Associated with these trajectories is a monotonic contribution to $R^{-1}(H)$, which falls off as $1/H$. The trajectories of type 2 can intersect S many times—on the order of $b\omega_c/v_\|$ times. As H is increased, the number of intersections increases, changing from an even number, when there is no contribution to the current, to an odd number, when there is a contribution to the current from intersections on strips of width $v_\|/\omega_c$ besides the edges of a hole of length a. Associated with these trajectories, in addition to the monotonic contribution to $R^{-1}(H)$, which falls off as $1/H$, is a contribution which oscillates along the H scale with a period $\Delta H = 2\pi cmv_F/eb$ and which falls off as $1/H^{5/2}$. These oscillations are reminiscent of the Sondheimer oscillations in the resistance of a film.[6] The oscillations should evidently be at a maximum at fields corre-

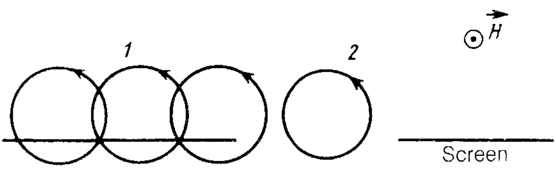

FIG. 1. Projection of various electron trajectories onto the plane perpendicular to H.

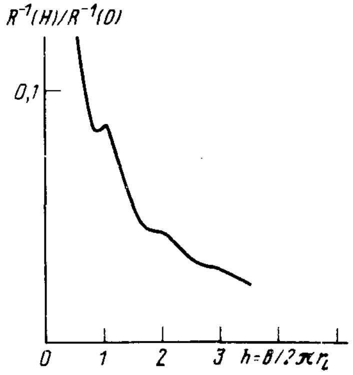

FIG. 2. Current through the point contact versus the magnetic field in the case $a \gg r_L$.

sponding to $h \simeq 1$. Figure 2 shows the results of a numerical calculation of $R^{-1}(H)/R^{-1}(0)$ versus H in this field region for the case $a \gg r_L$. The presence of the oscillations is manifested in the existence of regions where the curve flattens out abruptly, near integer values of h.

We turn now to the case of weak fields. The trajectories of type 1 contribute to the current over the entire plane S, just as they do in the case $H = 0$. Among the trajectories of type 2 we should distinguish between those with a small radius $v_\perp/\omega_c \simeq a \ll r_L$ and those with a normal radius on the order of r_L. The former have a spiral pitch $2\pi v_\parallel/\omega_c \approx 2\pi r_L \gg b$, intersect S once, and therefore also contribute to the current in precisely the same way as in the case $H = 0$. Magnetoresistance (3) is thus governed by trajectories of type 2 with a radius on the order of $r_L \gg a$. The relative number of such trajectories among all the trajectories which pass through a fixed point in the hole is on the order of a/r_L. The velocity V_n in (1) for these trajectories is on the order of $v_F(a/r_L)$. Consequently, the magnetoresistance is on the order of $(a/r_L)^2$, in agreement with (3) with $b \gg a$. At $b \ll a$, this estimate should be multiplied by the small factor b/a, since only electrons with a small $v_\parallel \sim v_F(b/a)$ can intersect hole S twice and therefore contribute to the magnetoresistance.

Since the oscillations in $R(H)$ stem from a geometric resonance, they may persist when diffuse scattering by the screen is taken into account.

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