Surface tension of helium II and extrapolation length for the order parameter

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Data on the temperature dependence of the surface tension of liquid ^{4}He near and below the λ-point can be used to obtain quantitative information on the extrapolation length l which figures in the condition $l d\Psi/dz = \Psi$ for the macroscopic wave function $\Psi = \sqrt{n_s}\, e^{i\varphi}$ at the boundaries of helium II. In the case of an interface between helium II and the vapor, this length is $l = 22.5 \pm 2$ Å; in the case of a boundary between liquid and solid helium it is $l \gtrsim 5$ Å.

Below the λ-point, the surface tension of liquid ^{4}He contains[1–3] an additional component σ_Ψ, which stems from the nonuniformity of the distribution of the order parameter—the macroscopic wave function $\Psi = \sqrt{n_s}\, e^{i\varphi}$—near the boundary. Recent experiments[4–8] have made it possible to identify the component $\sigma_\Psi \equiv \sigma_{\rm II}(T) - \sigma_{\rm I}(T)$ and to study its temperature dependence both near and far from the λ-point. It has been learned that the experimental results[4,5,8] agree with the theory[2] in the immediate vicinity of T_λ. At a separation as small as $t = (T_\lambda - T)/T_\lambda \gtrsim 10^{-2}$, one observes deviations from the functional dependence $\sigma_\Psi \sim t^{4/3}$ predicted in Ref. 2, and at low temperatures the component σ_Ψ is[7] an order of magnitude smaller than would follow

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from the simple theoretical estimate[1-3]

$$\sigma_\Psi = \frac{1}{3}\frac{\hbar^2}{m}\frac{n_{se}(T)}{\xi(T)} . \tag{1}$$

Here $n_{se}(T) = \rho_{se}(T)/m$ is the equilibrium density of the superfluid part in the interior of helium II, m is the mass of the ^{4}He atom, and $\xi(T) \equiv \xi_-(T)$ is the correlation length of the order parameter at $T < T_\lambda$ (near the λ-point we have $\xi_- = \xi_+/\sqrt{2} = \xi_0 t^{-2/3}, \rho_{se} = \rho_{s0} t^{2/3}$, and $\sigma_\Psi = \sigma_0 t^{4/3}$, where $\xi_0 = 1.15$ Å, $\rho_{s0} = 0.35$ g/cm^3, and $\sigma_0 = 0.254$ erg/cm^2; Ref. 3).

The discrepancies with the experimental data at low temperatures can be reduced by replacing $n_{se}(T)$ in (1) by $n_0(T)$—the density of particles in the condensate—as was suggested by Campbell.[9] If we do this, however, we end up with unrealistically large values of n_0 near the λ-point.[8] Furthermore, even at low temperatures the observed[7] behavior of the difference $\sigma_\Psi(0) - \sigma_\Psi(T)$ reflects not so much the temperature dependence of the condensate density, $n_0(0) - n_0(T) \sim T^2$, as that of the density of the superfluid part, $\rho_{se}(0) - \rho_{se}(T) \simeq \rho_{ne}(T) \sim T^4$. Finally, from the theoretical standpoint there is little justification for replacing n_s by n_0 in (1), at least near T_λ. The situation is that the replacement $\Psi \to \tilde{\Psi} = \sqrt{n_0} e^{i\varphi}$, i.e., a change in the normalization of Ψ, also requires a renormalization of the coefficient of the gradient term in the density of the thermodynamic $[(\hbar^2/2m) \to (\hbar^2/2m^*)$, where $m^* = mn_0/n_{se}]$, since otherwise the part of this term which depends on the phase φ would cease to have the meaning of a density of the kinetic energy of the superfluid motion. As a result, expression (1) would turn out to be independent of the method used to normalize Ψ. This point was disregarded in Ref. 9.

In the present letter we show that the discrepancy between the theory and the experimental data can be completely eliminated by abandoning the condition $\Psi = 0$ at the boundary [this condition was used in deriving (1)] and by replacing it by the more general boundary condition

$$d\Psi/dz = l^{-1}\Psi , \tag{2}$$

where the coordinate z is reckoned from the free surface in the direction into the He II, while l is a phenomenological length often called the "extrapolation length."

The need to use boundary condition (2) with increasing distance from T_λ was pointed out previously.[2,3,6] However, new data which have recently appeared[7,8] and attempts[7-9] to interpret them on the basis of the temperature dependence of the number density of the particles in the condensate make it worthwhile to move on at this point to a new and more comprehensive analysis.

Using (2), we modify expression (1) for σ_Ψ as follows[3]:

$$\sigma_\Psi \equiv \sigma_{II} - \sigma_I = \frac{\hbar^2}{2m}\frac{n_{se}(T)}{\xi(T)}\left\{\int_{\psi_0}^{1}(1-\psi^2)\sqrt{1+\nu_0^2\psi^2}\,d\psi + \frac{\xi(T)}{l}\psi_0^2\right\}. \tag{3}$$

Here $\psi = |\Psi|/\sqrt{n_{se}(T)}$, $\nu_0^2 = 2M/(M+3)$, $M > 0$ is a parameter of the theory,[3] and

ψ_0 is the value of the function ψ at the boundary. This value is found from condition (2) or, equivalently, from the condition for minimizing σ_Ψ with respect to ψ_0:

$$\frac{\partial \sigma_\Psi}{\partial \psi_0} \sim \left[2\frac{\xi(T)}{l}\psi_0 - (1-\psi_0^2)\sqrt{1+\nu_0^2\psi_0^2} \right] = 0. \tag{4}$$

It follows from (4) that near T_λ, where $\xi(T) \gg l$, the value is $\psi_0 \simeq l/2\xi(T) \ll 1$, and expression (3) converts (as it should) into (1), with the addition of a factor $K(\nu_0) \simeq 1 + 0.1\nu_0^2$, which reflects the weak dependence of σ_Ψ on ν_0, to the right side of (1).

On the contrary, far from T_λ, with $\xi(T) \ll l$, the value of ψ_0 at the boundary approaches the value $\psi = 1$ in the interior of the He II. In this case (3) is dominated by the second ("purely surface") term, and we have

$$\sigma_\Psi = \frac{\hbar^2}{2m}\,\frac{n_{se}(T)}{l}\,. \tag{5}$$

Expression (5) differs from (1) in that ξ has been replaced by the significantly greater length l. This replacement is, according to our assumption, related to the relatively small value of the components σ_Ψ at low temperatures and also the proportionality of the difference $\sigma_\Psi(0) - \sigma_\Psi(T)$ to the density of the normal part. According to the data of Ref. 7, we have $\sigma_\Psi(0) \simeq 20$ merg/cm^2. Also using (5), we conclude that at low temperatures the length is $\lambda = 9.1$ Å.

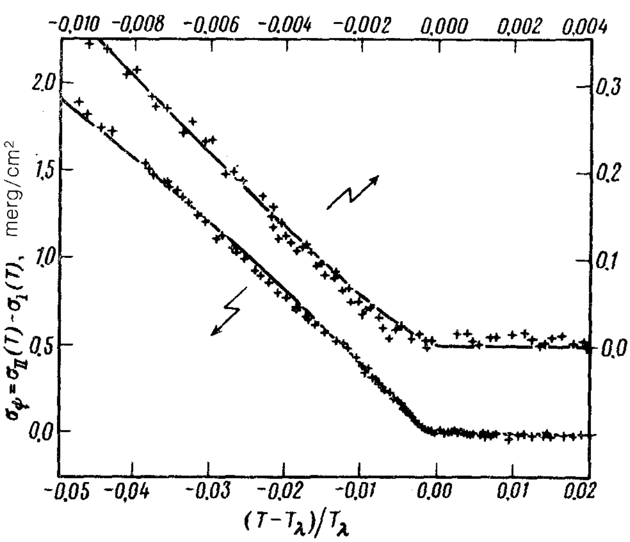

FIG. 1. Temperature dependence of the difference between the surface tension of helium II and that of helium I near the λ-point. The solid lines are calculations from expressions (3) and (4) with $l = 22.5$ Å and $M = 0.5$; the plus signs are experimental data.[8]

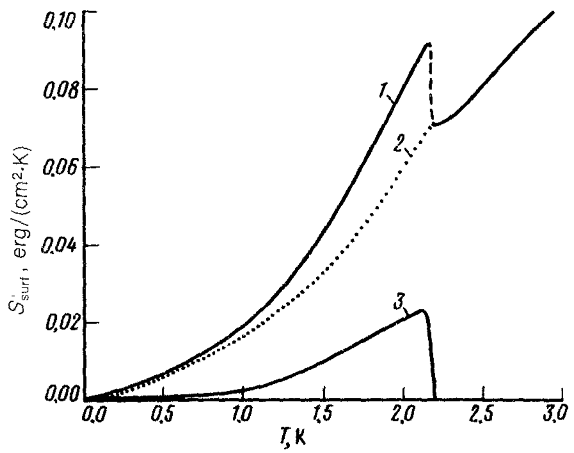

FIG. 2. Temperature dependence of the surface entropy S_{surf} of liquid ^{4}He below and near the λ-point. 1—Smoothed experimental curve[7]; 2—the same curve, after subtraction of the component S_Ψ calculated from (3) and (4) with $M=0.5$, $l(T_\lambda)=2.2$ Å, and $b=1$; 3—the component S_Ψ.

The primary distinctive features of the experimental results near T_λ are the nearly linear dependence $\sigma_\Psi(t)$ over the interval between $t \simeq 5\times10^{-2}$ and $t\simeq 5\times 10^{-3}$ (Fig. 1) and the presence of a clearly expressed maximum in the surface entropy $S_{surf} = -d\sigma/dt$ at $t\simeq 10^{-2}$ (Fig. 2). Both of these features can be explained quite easily in terms of a transition from dependence (5) to dependence (1) at $t\simeq 10^{-2}$. Since $n_s(T)$ is a concave function of the temperature, and $n_s(T)/\xi(T) \sim n_s^2(T)$ convex, the curve of $\sigma_\Psi(T)$ should have an inflection point at $\xi(T)\sim l$, i.e., precisely at $t\simeq(\xi_0/l)^{3/2}\simeq -10^{-2}$. The height of the maximum of S_Ψ near T_λ is related to the length l by

$$S_{\Psi,\, max} = -\left(\frac{d\sigma_\Psi}{dT}\right)_{max} = \frac{\sigma_0}{T_\lambda}\left(\frac{\xi_0}{l}\right)^{1/2} F(M),$$

where $F(0) = 1.03$ and $F(1) = 1.09$.

The best description of the experimental data near T_λ (Fig. 1) is reached with the parameter values

$$l = (22.5 \pm 2)\ \text{Å}, \qquad M = 0.5 \pm 0.3.$$

A good agreement with experiment over the entire temperature range T_λ can be reached by using the same values of l and M and by adding to (3) a term $b[\xi(T)n_{se}(T)/l]\psi_0^4$ or by simply using the following temperature dependence of the length l:

$$l = l(T_\lambda)/(1 + bn_{se}(T)/n_{se}(0)) \quad \text{with } b\simeq 1 \quad \text{(Fig. 2)}.$$

The nonmonotonic behavior of the surface entropy near T_λ which remains after the component S_Ψ is subtracted (curve 2 in Fig. 2) can be attributed to long-wave fluctuations,[10] which are not considered in the theory of Ref. 3, or to a nonmonotonic temperature dependence of the component of S_{surf} which comes from the profile of the ion density.[4]

In principle, the value of the parameter l might be affected by ^{3}He impurities. Furthermore, condition (2) with generally different values of l can also be used for the interface between He II and the solid. It follows from the available experimental data,[11] which are not yet comprehensive, that the length for the case of the interface between He II and solid ^{4}He is $l \gtrsim 5$ Å.

It is important to consider the difference between the parameter l and zero in calculations of the shift of the λ-point and of other size effects in helium films and layers of thickness $d \lesssim l$. Finally, we note that a similar approach might be taken to analyze the temperature dependence of the surface thermodynamic quantities in the case of any second-order phase transitions, e.g., in the case of transitions to a superfluid state in liquid ^{3}He or in the case of transitions from isotropic to liquid-crystal phases.

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