Low-frequency dynamics of systems with 2D electronic layers in a magnetic field

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Equations for macroscopic dynamics of dielectric systems containing a 2D conducting layer in an external magnetic field are obtained. The rotation of the polarization plane of the transverse acoustic and electromagnetic waves and the mutual conversion of these waves in such a layer are observed. The low-frequency magnetoplasma waves in a dielectric sandwich with 2D conducting channels are studied.

Since the discovery of quantization of the Hall effect in 2D electronic layers at low temperatures in a strong magnetic field, an interest in the study of dynamic low-frequency phenomena in such systems has increased considerably. The quantization of the Faraday effect in the case of transmission of radio waves through a layer, for example, was predicted by Volkov and Mikhaĭlov[1] and observed experimentally by Volkov *et al.*[2] A low-frequency magnetoplasma resonance with a frequency inversely proportional to the external magnetic field was observed in Refs. 3 and 4 in a semiconductor system with a 2D conducting channel in a strong magnetic field. There is as yet, however, no systematic macroscopic description of the dynamic low-frequency phenomena in these systems. In this study we obtained equations for the electrodynamics and elastic dynamics of the dielectric systems (or semiconductor systems at low temperatures) with 2D conducting layers in an external magnetic field. We also considered certain problems which are directly related to the experiments on low-frequency dynamics of such systems.

The equations for the dynamics of these systems can be reduced to Maxwell's equations, to elasticity theory (hydrodynamics) equations for contiguous dielectric media, and to the boundary conditions of these equations for the conducting-layer plane. The macroscopic boundary conditions are

$$\tau_{ni}^{(1)} - \tau_{ni}^{(2)} = \frac{1}{c}[\mathbf{j}_S, \mathbf{H}_0]_i, \qquad \dot{u}_{i1} = \dot{u}_{i2},$$
$$[\mathbf{H}_1 - \mathbf{H}_2, \mathbf{n}] = \frac{4\pi}{c}\mathbf{j}_S, \qquad E_{\mu 1} = E_{\mu 2}, \tag{1}$$
$$\mathbf{j}_S = \hat{\sigma}_S\left(\mathbf{E} + \frac{1}{c}[\dot{\mathbf{u}}, \mathbf{H}_0]\right),$$

where τ_{ik} is the tensor of the bulk elastic strain, u_i is the elastic-displacement vector, $\tau_{ni} = \tau_{ik} n_k$, $\mathbf{n}$ is the unit vector of the normal to the boundary which is directed from medium 1 to medium 2, the subscripts $\mu,\nu = 1,2$ specify the coordinate axes in the tangential plane, $\mathbf{j}_S$ and $\hat{\sigma}_S = \sigma_{\mu\nu}(\omega, H_0)$ are the current density and the dynamic-conductivity tensor (per unit area) of the 2D conducting layer, and $\mathbf{H}_0$ is the external magnetic field (from now on, $\mathbf{H}_0$ is assumed to be directed along the normal to the layer). We wish to emphasize that Eqs. (1) can be used not only for 2D conducting inversion layers in MIS structures or heterostructures[5] but also for macroscopically thin metal films of thickness less than the wavelength and skin-layer depth in them, deposited on the dielectric (or semiconductor) surface. At low temperatures and strong magnetic fields such metallic films can be used as effective electromagnetic-acoustic converters.[6] The system of equations (1) can also be used to describe the magnetohydrodynamic and electrodynamic phenomena in a 2D electronic layer on the surface of liquid[7] ^{4}He in a strong magnetic field.

Let us consider the rotation of the polarization plane of the electromagnetic (EM) waves and transverse acoustic waves transmitted through an electronic layer (the Faraday effect and its acoustic analog) and let us also consider the mutual conversion of these waves in a solid system. In the case of an incident electromagnetic wave, a calculation of the reflection and transmission coefficients shows that, in contrast with an electromagnetic wave reflected from a 3D conducting medium,[8] the interaction of such a wave with a 2D conducting layer does not lead to an elliptic polarization of reflected (or transmitted) low-frequency waves. If the Hall effect is quantized in the conducting inversion layers, where $\sigma_{xx} \ll |\sigma_{xy}|$, $\sigma_{xx} \ll c(\sqrt{\epsilon_1} + \sqrt{\epsilon_2})$, and $\omega \ll \omega_c = eH_0/mc$, the polarization plane of the transmitted electromagnetic wave is virtually the same as that of the transverse excited acoustic wave: $\theta_{\mathrm{EM}} \approx \theta_{\mathrm{EMA}} \approx 4\pi\sigma_{xy}/c(\sqrt{\epsilon_1} + \sqrt{\epsilon_2})$. In the opposite limiting case $\sigma_{xx} \gg |\sigma_{xy}|$, $\sigma_{xx} \gg c(\sqrt{\epsilon_1} + \sqrt{\epsilon_2})$ (which is realized, for example, in macroscopically thin metallic films in weak magnetic fields, $\omega_c\tau \ll 1$), on the other hand, these planes are nearly orthogonal: $\theta_{\rm EMA} \approx \pi/2$, $\theta_{\rm EM} \approx \sigma_{xy}/\sigma_{xx} \ll 1$. Here $\theta_{\rm EM}$ and $\theta_{\rm EMA}$ are the tilt angles of the polarization plane of the transmitted electromagnetic wave and transverse elastic wave relative to the polarization plane of the electric field of the electromagnetic wave incident along the normal to the layer, $\sigma_{xy} = -\sigma_{yx}$ and $\sigma_{xx} = \sigma_{yy}$ are the Hall conductivity and the dissipative conductivity of the electronic layer, and ϵ_1 and ϵ_2 are the dielectric constants of the contiguous media.

The conversion coefficient of the conversion of an electromagnetic wave into an elastic wave (the ratio of the energy flux of the transmitted acoustic wave to that of the incident electromagnetic wave) in the 2D conducting layer does not depend on the frequency. In the limiting case of a vanishingly small dissipative conductivity σ_{xx} we find

$$K = 16\pi \frac{\rho_2 c_{t2}\sqrt{\epsilon_1}(\sigma_{xy}H_0)^2}{c^3(\sqrt{\epsilon_1}+\sqrt{\epsilon_2})^2(\rho_1 c_{t1}+\rho_2 c_{t2})^2} \sim \pi \frac{(\sigma_{xy}H_0)^2}{\rho c_t c^3\sqrt{\epsilon}}\,; \tag{2}$$

i.e., the conversion coefficient K does not depend on the external magnetic field (ρ and c_t are the density and velocity of the transverse sound in the contiguous media; the electromagnetic wave from medium 1 is the incident wave). In the case of a large dissipative conductivity (see the discussions above), on the other hand, for K we find

$$K = 16\pi \frac{\rho_2 c_{t2}\sqrt{\epsilon_1}H_0^2}{c(\rho_1 c_{t1}+\rho_2 c_{t2})^2} \sim 4\pi \frac{H_0^2\sqrt{\epsilon}}{\rho c c_t}\,; \tag{3}$$

i.e., the conversion coefficient is proportional to H_0^2 and is virtually independent of the conductivity of the electronic layer.

The transverse elastic wave incident on a 2D conducting layer was studied in a similar manner: we found the rotation of the polarization plane and the conversion to an electromagnetic wave. For the angle of rotation θ_A of the polarization plane of a transverse acoustic wave transmitted along the normal to the layer we find

$$\tan\theta_A = \frac{\sigma_{xy}H_0^2(\sqrt{\epsilon_1}+\sqrt{\epsilon_2})^2}{(\rho_1 c_{t1}+\rho_2 c_{t2})[(4\pi\sigma_{xy})^2+(4\pi\sigma_{xx}+c\sqrt{\epsilon_1}+c\sqrt{\epsilon_2})^2]}\,;$$

i.e., at $\omega_c\tau \gg 1$ the rotation angle is proportional to H_0. In the limiting cases considered above, the conversion factor of the conversion of an elastic wave into an electromagnetic wave has a value approximately equal to that in (2) or (3), respectively (the principle of reciprocity).

Let us now consider the electromagnetic waves in a sandwich structure consisting of two boundless dielectric plates, ϵ_1 and ϵ_2, of thickness d_1 and d_2, between which a 2D conducting channel is situated. In the simplest case, $\epsilon_1 = \epsilon_2$ and $d_1 = d_2 = d$, the dispersion equation for the branch of the electromagnetic waves which interact with the electronic layer at $\omega \ll \omega_c$ and $\sigma_{xx} \ll |\sigma_{xy}|$ has the form

$$\frac{\frac{i\kappa_1}{\kappa_0}\tan\kappa_1 d - 1}{\tan\kappa_1 d + \frac{i\kappa_1}{\kappa_0}} \quad \frac{\tan\kappa_1 d + i\frac{\kappa_1\epsilon_0}{\kappa_0\epsilon_1}}{1 - i\frac{\kappa_1\epsilon_0}{\kappa_0\epsilon_1}\tan\kappa_1 d} = \frac{1}{\epsilon_1}\left(\frac{2\pi\sigma_{xx}}{c}\right)^2 , \tag{4}$$

where $\kappa_{1,0} = (\epsilon_{1,0}\omega^2/c^2 - k^2)^{1/2}$, and ϵ_0 is the dielectric constant of the external medium. If a plate with a large dielectric constant, $\epsilon_1 \gg 1$, is situated in a vacuum ($\epsilon_0 = 1$), the spectrum of its lowest-frequency branch (with a zero critical frequency) at $1/\epsilon_1 \ll kd \ll 1$ will have the form

$$\omega^2 = \frac{c^2 k}{\epsilon_1 d} + \left(\frac{2\pi\sigma_{xy}}{\epsilon_1 d}\right)^2 , \qquad 2\pi|\sigma_{xy}| \ll c\sqrt{\epsilon_1'} . \tag{5}$$

If the plate under consideration contains N Hall conducting layers (the quantum superlattice), the quantity σ_{xy} in Eq. (5) [and Eq. (6)—see the text below] should be replaced by $N\sigma_{xy}$. If the condition $1 \ll 2\pi N|\sigma_{xy}|/c \ll \sqrt{\epsilon_1'}$ is satisfied, the system will exhibit circularly polarized uniform ($kd \ll 1$) magnetoplasma electric-field oscillations in the plane of the layers, with a frequency $\omega_{\rm MP} = 2\pi N(|\sigma_{xy}| - i\sigma_{xx})/\epsilon_1 d$. The magnetic field of these waves is small in comparison with the electric field. These electromagnetic waves, which have a waveguide nature (the phase velocity of the waves is greater than the velocity of light in the medium, $\omega > \cos k/\sqrt{\epsilon_1}$), do not reduce to electrostatic waves in bounded quantum superlattices,[3] but they are nonetheless caused by the Hall dissipation-free conductivity of the 2D channels.[1)] If the outer surfaces of the dielectric sandwich are metallized surfaces ($\epsilon_0, \kappa_0, \epsilon_0/\kappa_0 \to \infty$), Eq. (4) becomes $\cot^2\kappa_1 d = (2\pi\sigma_{xy}/c\sqrt{\epsilon_1})^2$. At $2\pi|\sigma_{xy}| \ll c\sqrt{\epsilon_1}$, $k = 0$ this equation can then be used to find

$$\omega_n(0) = \left(\frac{\pi}{2} + \pi n\right)\frac{c}{d\sqrt{\epsilon_1}} \pm \frac{2\pi\sigma_{xy}}{\epsilon_1 d} , \qquad n = 0, 1, \ldots \tag{6}$$

At $\omega \ll \omega_c$ and $\omega_c\tau \gg 1$ the external magnetic field thus leads to a shift (and a splitting) of the frequencies of the waveguide electromagnetic waves in a dielectric sandwich with 2D electronic layers. This shift is inversely proportional to the magnetic field and thickness of the sandwich.

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[1)]The existence of such a nonelectrostatic electromagnetic resonance in a boundless quantum superlattice with a frequency $\omega_0 = 4\pi(|\sigma_{xy}| - i\sigma_{xx})/eh$ (h is the distance between the Hall 2D conducting layers) is implied by the equations for low-frequency electrodynamics of such systems.[9]

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