Fractal behavior in multiple production

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A method is proposed for determining the internal dimensionality of the random walks of partons from experimental data on the average multiplicity and slope of the diffraction cone in hadronic processes. A comparison is made with e^+e^- annihilation.

The space-time characteristics of the systems which are produced as a result of high-energy collisions of two hadrons have been under discussion since the early studies by Heisenberg and, later, by Fermi, Pomeranchuk, and Landau on statistical and hydrodynamic descriptions of the evolution of such systems. These characteristics were subsequently studied on the basis of multiperipheral (cluster and parton) models and also for a quark-gluon plasma. There is also obvious interest in the space-time evolution of a parton cascade in electron-positron annihilation and in deep inelastic production of hadrons.

The process of multiple production of hadrons is usually considered to go through a first stage of the production of a large number of partons (quarks and gluons), which then convert into final colorless hadrons as a result of interactions, the directed motion, diffusion, and adhesion. There are several possibilities for a phenomenological description of the dynamics of the process: a multiperipheral description (with rescattering), a string description (in which the hadronization of the strings is a classical stochastic process), and a cascade description (the Altarelli-Parisi equations).

In particular, in a simple multiperipheral ladder, the development of the process is represented as a Brownian motion of partons[1,2] between particle production points. The mean-square value of the impact parameter, $\bar{\rho}^2$ (a distance in the transverse plane), which determines the slope of the diffraction cone, b, is proportional to the number of steps, i.e., to the average multiplicity $\bar{n}$:

$$b \sim \bar{\rho}^2 \sim \bar{n}, \tag{1}$$

where $\bar{n}$ and b increase logarithmically with the energy in the multiperipheral picture. Actually, their growth takes different forms experimentally and is significantly faster for $\bar{n}$. Such a simple relationship between the slope of the cone and the average multiplicity is disrupted even in the multiperipheral picture if the rescattering of the initial partons and the interaction of partons in the ladder are taken into account. The diffusion of partons is even more complicated, and the diagrams themselves become reminiscent of branched electric circuits or fractals.[3–5]

We should naturally like to know the internal dimensionality of the random walks of partons in the system that is formed. This dimensionality, D'_W, is found from[5]

$$b \sim \overline{\rho^2} \sim \bar{n}^{2/D'_W} \quad , \tag{2}$$

where $D'_W = 2 + \theta$, and θ is the anomalous-diffusion exponent [the diffusion coefficient has a distance dependence $\kappa(\rho) = \kappa\rho^{-\theta}$]. We then find

$$D'_W = 2\frac{d\ln\bar{n}}{d\ln b} \, . \tag{3}$$

In other words, by working from experimental data on the multiplicity and the slope of the diffraction cone we can determine the internal dimensionality of the random walks of the partons. Figure 1 shows some corresponding results for the pp and $p\bar{p}$ interactions. We see that in a first approximation these results can be described by a common straight line, from whose slope we find

$$(D'_W)_p \approx 7.5 \pm 1.5 \, . \tag{4}$$

Such a large value (by way of comparison, for Brownian motion we would have $\theta = 0$ and $D'_W = 2$) for the internal dimensionality of a random walk[1)] indicates that the parton diffusion of the parton involves many returns, i.e., that the path of the parton is an extremely intricate trajectory. The probability for finding the parton with an impact parameter ρ has the behavior

$$P \sim \exp\left[-\frac{\rho^{D'_W}}{\kappa D'^2_W \bar{n}}\right]. \tag{5}$$

Experimental data on πp at lower energies yield $(D'_W) \approx 4$.

There is another way to interpret these results. It may be that it is not the intricacy of the trajectory of an individual parton that is increasing but the number of partons. The slope of the cone reflects the average remoteness of the parton. In the case of a Brownian motion, the number of steps of a parton should then be $\bar{n}^{2D'_W}$; i.e., the number of partons would be large, $\sim \bar{n}^{1-2/D'_W}$.

Regardless of how they are interpreted, the experimental data do provide an argument in favor of statistical-hydrodynamic models for inelastic hadron collisions.

Let us attempt to apply ideas regarding a random walk of partons to e^+e^- annihilation. Unfortunately, we do not have at our disposal such a quantity as the slope of

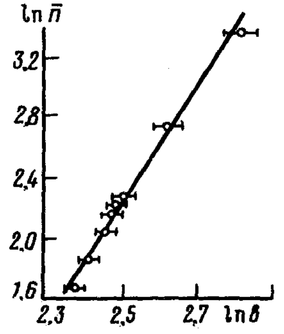

FIG. 1. The experimental data on the average multiplicity $\bar{n}$ and the slope of the diffraction cone, b, for pp and $p\bar{p}$ interactions (the points) determine the dimensionality of the internal random walk of a parton. The slope of the line drawn here yields the value $D'_W \approx 7.5 \pm 1.5$.

the diffraction cone, and the only way we can determine the internal dimensionality of the random walk of partons is to work from a specific theoretical model. As this model we adopt the Altarelli-Parisi equations,[6] introducing a dynamic cutoff of the cascade development.[7,8] Here the role of the square of the distance is played by the inverse square of the 4-momentum of the parton, $1/k^2$. It thus seems natural to determine the internal dimensionality of the random walk of partons from the following expression, by analogy with (3):

$$D'_W = -2\frac{d\ln \bar{n}}{d\ln k^2}. \tag{6}$$

Using the solutions of the equations for the development of a gluon cascade from an initial gluon which were found in Refs. 5 and 6, we find[2)]

$$D'_W = \frac{e^{2\pi b y}}{\pi b \ln Q^2/\Lambda^2}\left[\frac{I_1(\sqrt{8C_V y \ln E_0/\beta})}{I_0(\sqrt{8C_V y \ln E_0/\beta})}\left(\frac{2C_V \ln E_0/\beta}{y}\right)^{1/2} - a\right], \tag{7}$$

where $b = 33 - 2n_f/12\pi$; $n_f = C_V = 3$; $a = 101/18$; $y = (1/2\pi b)\ln(\ln Q^2/\Lambda^2)/(\ln k^2/\Lambda^2)$; E_0 is the initial energy; Q^2 is the initial square 4-momentum of the parton; Λ and β are cutoff parameters, which we assume to be identical; and I_μ is the Bessel function of order μ.

At $y = 0$, i.e., at the very beginning of the evolution of the cascade, we thus find

$$D'_W = \frac{1}{\pi b \ln Q^2/\Lambda^2}\left(2C_V \ln\frac{E_0}{\beta} - a\right). \tag{8}$$

Assuming $Q^2 \sim E_0^2/4$, we find the estimate $D'_W \sim 1$ (as $E_0 \to \infty$, we have $D'_W = 4/3$); i.e., the parton deviates only slightly from a straight line during the initial stage of the development of the cascade. Using the experimental energy dependence of the multiplicity (Ref. 9, for example), we can estimate the end of the development of the cascade, y_c, from

$$(8C_V y_c)^{1/2} = 1.92 \pm 0.07. \tag{9}$$

Substituting this value of y_c into (6), we easily see that at the end of the development of the cascade the value of D'_W is not greatly different from unity, and it varies only slightly at today's energies. We thus conclude that the partons deviate only slightly from a straight line during the development of the cascade in electron-positron annihilation, in contrast with the picture of a complicated, intricate motion as in hadron collisions.

The method proposed here could of course be applied to other processes in which a parton interpretation is possible. Unfortunately, this method tells us nothing about the fractal dimensionality D of the system that is formed,[3)] but a high dimensionality of an internal random walk of partons in a hadronic system would presumably imply that this system has a complex structure.

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[1)]Although D'_W cannot be determined highly accurately because of uncertainties in the determination of b (actually, values which lead to a lower boundary on D'_W have been chosen), it is important to note that this quantity is significantly greater than 2.

[2)]Other cascades differ only in the coefficient of the exponential functions, which are inconsequential here.

[3)]It appears only in the coefficient of the exponential function in solution (5) of the diffusion equation.[5,10]

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