The strength function of germanium-71 and the problem of the $\nu_e \leftrightarrow \tilde{\nu}_e$ oscillations

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The strength function and the cross section of the reaction ${}^{71}\text{Ga}(\nu_e, e^-){}^{71}\text{Ge}$ are calculated. In connection with the study of the $\nu_e \leftrightarrow \tilde{\nu}_e$ oscillations, it is shown that allowance for the transitions to the excited states of ${}^{71}\text{Ge}$ leads to an approximately twofold increase of the total cross section in comparison with the cross section of the transition between the ground states of ${}^{71}\text{Ga}$ and ${}^{71}\text{Ge}$.

An experiment in which the difference between ν_e and $\tilde{\nu}_e$ can be studied was proposed in Ref. 1. The aim of this experiment was to detect the process

$$\tilde{\nu}_e + {}^{71}\text{Ga} \rightarrow {}^{71}\text{Ge} + e^- \tag{1}$$

in the flux of $\tilde{\nu}_e$ from a high-power nuclear reactor.

Although the accuracy of experiments on the search for the $2\beta(0\nu)$ decay, which also have as their goal the study of the difference between ν_e and $\tilde{\nu}_e$, has recently increased markedly, the study of reaction (1) may yield new and independent information, especially in the region of the $\nu_e \leftrightarrow \tilde{\nu}_e$ oscillations.

The degree of overlapping of the ν_e and $\tilde{\nu}_e$ states is usually characterized by $\alpha^2 = \sigma_{\text{exp}}/\sigma_{\text{calc}}$, i.e., by the ratio of the measured cross section for reaction (1) to the calculated cross section under the assumption that $\tilde{\nu} \equiv \nu_e$ (see, e.g., Ref. 14).

In Ref. 1 the calculated cross section σ_{calc} was obtained for the transition to the ground state of ${}^{71}\text{Ge}$. In the present letter we estimate the total cross section σ_{calc} on the basis of a calculation of the strength function, with allowance for the excited states of the ${}^{71}\text{Ge}$ nucleus.

The cross section $\sigma(E_\nu)$ for reaction (1) is given by

$$\sigma(E_\nu) = \frac{g_A^2}{\pi c^3 \hbar^4} \int_0^{E_\nu - Q} p_e E_e F(Z, E_e) S_\beta(E') dE', \tag{2}$$

where g_A is the axial-vector coupling constant, p_e and E_e are the momentum and energy of the electron, $F(Z,E_e)$ is the Fermi function, Q is the threshold energy, $S_\beta(E')$ is the strength function which is defined as the reduced transition probability per unit energy, and $E_e = E_\nu - Q - E' + m_e c^2$. Except for the strength function, all the terms in Eq. (2) are known with sufficient accuracy. There are examples in the literature of the calculation of the strength function on the basis of different models.[2,3] In this study we have calculated the strength function with use of the theory of finite Fermi systems.[4] We solved the problem involving the eigenfunctions φ and eigenvalues ω for the effective β field with allowance for only the allowed transitions. We used the local interaction $\vec{\tau}_1\vec{\tau}_2$ and quasiparticle interaction $\vec{\tau}_1\vec{\tau}_2\vec{\sigma}_1\vec{\sigma}_2$ which was parametrized by the constants f_0' and g_0'. In the calculation we used the ratio $g_0'/f_0' = 0.93$ obtained in Ref. 5 from the analysis of experimental data on the Gamow-Teller resonance. The solutions of ω gave the isobaric-state spectrum over a broad interval from the ground state to the Gamow-Teller resonance. The residues of φ determined the matrix elements of the transition from the ground state of ^{71}Ga to the excited isobaric states of ^{71}Ge.

In the construction of the strength function $S_\beta(E)$ we took into account the fragmentation of the higher-lying isobaric states due to the multiparticle configurations, which led to their broadening. This broadening was calculated from the Breit-Wigner formula, as in Ref. 6. According to Migdal,[4] the width was chosen in the form

$$\Gamma(E') = \alpha(E')^2 + \beta(E')^3 + \dots\ . \tag{3}$$

In the calculations of S_β (E) it is sufficient to use only the first term of the expansion with $\alpha \approx \epsilon_F^{-1}$, which takes into account the three-quasiparticle configurations. We used the value $\alpha = 0.018$ obtained from the averaged experimental widths of the Gamow-Teller resonance.[7] The described method of calculating S_β was used previously to predict the structure of $S_\beta(E)$ for the rubidium isotopes[6] and the probability for the emission of several delayed neutrons for several nuclei.[8] These predictions were subsequently confirmed experimentally.

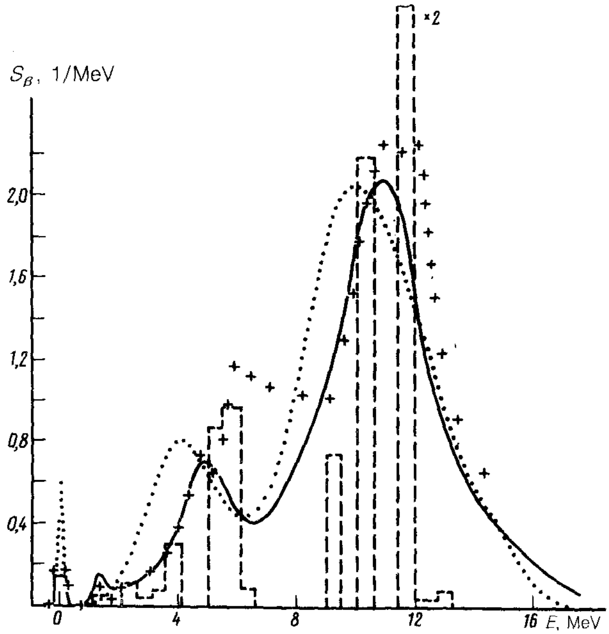

FIG. 1. The strength function of ^{71}Ge ($q = 0.5$). Solid curve—the results of our study; dashed curve—the results of Ref. 3; dotted curve—the results of Ref. 2; crosses— experimental data from the ^{71}Ga (p, n) ^{71}Ge reaction.[13]

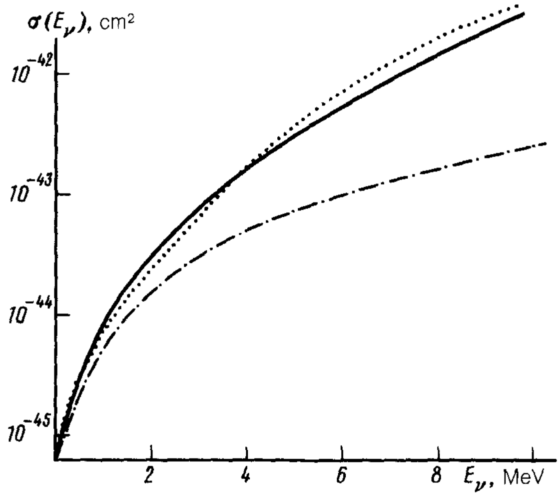

FIG. 2. The cross section of the reaction $^{71}\mathrm{Ga}(\nu_e, e^-)^{71}\mathrm{Ge}$ plotted as a function of the neutrino energy. Solid curve—calculation based on the strength function obtained by us; dotted curve—calculation based on the strength function obtained in Ref. 2; dot-dashed curve—cross section of the transition between the ground states $^{71}\mathrm{Ga}_{gs} \to {}^{71}\mathrm{Ge}_{gs}$.

The results of the calculation of S_β for ^{71}Ge are shown in Fig. 1. In the structure of the strength function we can clearly distinguish three isobaric states with energies $E_1 = 1$–2 MeV, $E_2 = 4$–5 MeV, and $E_3 = 10$–11 MeV, two of which with the energies E_3 and E_2 are the Gamow-Teller-resonance state and the collective isobaric state such as the core polarization.[5] To describe the ground state and the first two excited states, we used the well-known estimates, as in Refs. 2 and 3.

The sensitivity of the cross section $\sigma(E_\nu)$ to the choice of the strength function was determined from the calculations based on the strength functions of Refs. 2 and 3. All the strength functions of the Gamow-Teller transitions were normalized to $3(N-Z)/2$; i.e., it was assumed that the "quenching" is 0.5. The results of the calculations based on these strength functions are shown in Fig. 2.

The total interaction cross section can be found by contracting $\sigma(E_\nu)$ with the reactor-antineutrino spectrum (under the assumption that $\nu_e \equiv \tilde{\nu}_e$). There are several versions of the experimental spectra[9,10] and theoretical spectra[11,12] of reactor antineutrinos. We have carried out calculations for all spectra listed above, having thus determined the sensitivity of the cross section to the change in the antineutrino spectrum. In contrast with the chlorine-argon detector, the gallium-germanium detector has a low sensitivity threshold (235.7 keV), and the uncertainty associated with the low-energy part of the antineutrino spectrum should affect the result. To estimate the effect of this factor, we have varied the low-energy part of the spectrum in the range from zero to two $\tilde{\nu}_e$/MeV.

The results of calculations showed that the total cross section depends only moderately on the strength function that is used and on the reactor-antineutrino spectrum and its initial part, with the exception of the theoretical spectrum.[12] The cross section fluctuates between 71.3×10^{-45} cm^2/division for the strength function obtained in Ref. 2 and the spectrum obtained in Ref. 10 and 91.5×10^{-45} cm^2/division for the strength function obtained by us and the spectrum obtained in Ref. 9. This difference stems primarily from the presence of a peak at 1.31 MeV in our strength function and its absence in the strength function of Ref. 2. The ratio of the total cross section to the cross section for transition to the ground state also varies within 25%, from 2.1 to 2.6, and the principal contribution to this quantity produces a strength-function peak over the range 4-5 MeV. The cross section was found to depend only slightly on the spin-isospin coupling constant g_0': a 10% variation of g_0' corresponds to a 3% variation of the total cross section.

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[1] I. R. Barabanov, A. A. Borovoĭ, V. N. Gavrin, *et al.*, Preprint IYaI, P 0466, 1986.
[2] G. J. Mathews, S. D. Bloom, G. M. Fuller, and J. N. Bahcall, Phys. Rev. C **32**, 3 (1985).
[3] H. V. Klapdor and K. Grotz, MPI-1985, V9.
[4] A. B. Migdal, Theory of Finite Fermi Systems, Nauka, Moscow, 1983.
[5] Yu. V. Gaponov and Yu. S. Lyutostanskiĭ, Fiz. Elem. Chastits At. Yadra **12**, 1324 (1981) [Sov. J. Part. Nucl. **12**, 528 (1981)].
[6] Yu. S. Ljutostansky and I. V. Panov, Z. Phys. **A313**, 235 (1983).
[7] Yu. S. Ljutostansky, V. K. Sirotkin, and I. V. Panov, Phys. Lett. **161B**, 9 (1985).
[8] V. G. Aleksankin, Yu. S. Lyutostansky, and V. K. Sirotkin, Neutron Physics **3**, Moscow, 1980, pp. 124–128.
[9] P. Vogel *et al.*, Phys. Rev. C **24**, 1543 (1981).
[10] H. V. Klapdor and J. Metzinger, Phys. Lett. **112B**, 22 (1982).
[11] F. T. Avignone, Phys. Rev. D **2**, 2609 (1970).
[12] F. T. Avignone and Z. D. Greenwood, Phys. Rev. C **22**, 594 (1980).
[13] D. Krofcheck *et al.*, Phys. Rev. Lett. **55**, 1051 (1985).
[14] J. N. Bahcall and H. Primakoff, Phys. Rev. D **18**, 3463 (1978).

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