πK dimesonic atom

G. V. Efimov, M. A. Ivanov, and V. E. Lyubovitskiĭ
Joint Institute for Nuclear Research

(Submitted 23 March 1987)
Pis'ma Zh. Eksp. Teor. Fiz. **45**, No. 11, 526–529 (10 June 1987)

The properties of a πK dimesonic atom are discussed in connection with planned experiments at the Pozitroniĭ installation (Institute of High-Energy Physics and Joint Institute for Nuclear Research). The basic characteristics of the πK dimesonic atom—its lifetime, its binding energy, Lamb shift, and the value of the wave function at the origin—are calculated on the basis of the virton-quark model.

Experiments are being planned[1] to observe relativistic πK dimesonic atoms and to measure their basic characteristics. Measurements of their lifetime τ, the value of the wave function at the origin, $\Psi_{1S}(0)$, and the Lamb shift ΔE_{2S-2P} are planned. The lifetime and the value of the wave function at the origin are related by[2]

$$1/\tau \sim |a_0^{1/2} - a_0^{3/2}|^2 \, |\Psi_{1S}(0)|^2 ,$$

where $a_0^{1/2}$ and $a_0^{3/2}$ are the corresponding S-wave lengths for πK scattering. The proposed experiments will thus provide a model-independent determination of the difference between the S-wave lengths, $a_0^{1/2} - a_0^{3/2}$. A point which should be emphasized is that the πK scattering lengths are presently determined by approximating the energy dependence of the scattering phase shifts in the reaction[3] $KN \to K\pi + \ldots$. That procedure is afflicted by uncertainties of a model nature and results in large errors ($\sim 25\%$).

From the theoretical standpoint, a knowledge of πK scattering lengths can serve as a test of the hypotheses of various models which have been offered as candidates for a description of low-energy physics. For example, the πK scattering lengths and, correspondingly, the lifetime of the πK dimesonic atom were calculated in a nonlinear chiral theory by Bel'kov *et al.*[4] and in a superconducting quark model by Volkov and Osipov.[5] It turned out that the difference $a_0^{1/2} - a_0^{3/2}$ was independent of the mechanism responsible for the breaking of chiral symmetry. A measurement of the lifetime of the πK dimesonic atom thus serve as an independent test of the basic positions of chiral symmetry.

Information of a more profound nature on the structure of the πK dimesonic atoms could be found by studying its energy levels and wave functions. These properties are determined by both strong and electromagnetic interactions.

The contribution of strong interactions can be taken into account by means of Deser's formula[6]:

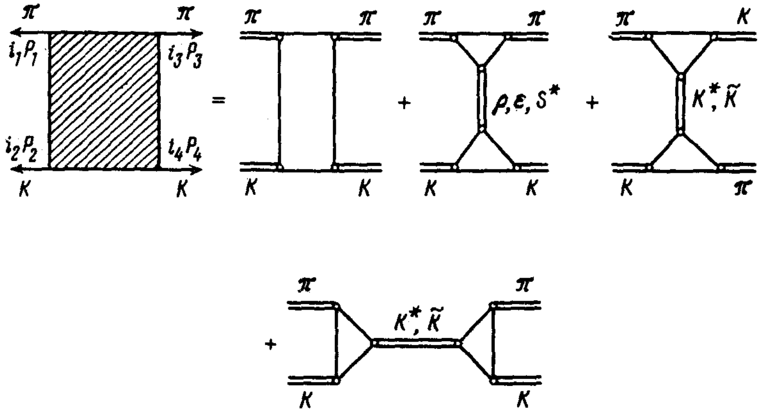
$$\Delta E_{nl}^{S} \sim (2a_0^{1/2} + a_0^{3/2}) \, |\Psi_{nl}^{cul}(0)|^2 ,$$

$$\Delta \Psi_{1S}^{S}(0) \sim (2a_0^{1/2} + a_0^{3/2}) \frac{|\Psi_{2S}^{cul}(0)|^2}{E_{1S}^{cul} - E_{2S}^{cul}} \Psi_{1S}^{cul}(0),$$

where E_{nl}^{cul} and Ψ_{nl}^{cul} are the Coulomb energy levels and wave function.[7]

Radiation and polarization contributions have been calculated by Karimkhodzhaev and Faustov[8] with the help of a quasipotential equation.

In the present letter we use the virton-quark model[9–11] to calculate the basic characteristics of the πK dimesonic atom: its lifetime, the value of the wave function at the origin, and the shifts of the energy levels due to the strong and electromagnetic interactions.



The interaction Lagrangian of the virton-quark model, which we will need in order to describe πK scattering, is[10]

$$\mathcal{L}_I(x) = g_M \bar{q}_a(x)\varphi_M(x)\lambda_M \Gamma^M q_{\bar{a}}(x),$$

where

$$q_{\bar{q}} = \begin{bmatrix} \bar{u}_{\bar{a}} \\ d_{\bar{a}} \\ S_{\bar{a}} \end{bmatrix}$$

is the quark field ($\bar{a}$ is the color index), φ_M is the meson field, g_M is the constant of the meson-quark interaction, λ_M is the Gell-Mann matrix, and Γ^M is the Dirac matrix. Figure 1 shows Feynman diagrams describing πK scattering. The corresponding invariant amplitude is written in the form[4]

$$T_{i_1 i_3,\, i_2 i_4}(P_1 \ldots P_4) = \delta_{i_1 i_3}\delta_{i_2 i_4} T^+(Stu) - \omega_{i_1 i_3}\tau_{i_2 i_4} T^-(Stu),$$

$$T^+(Stu) = -C_\square(Stu) - C_\square(utS) + \frac{C^2_{\tilde{K}\pi K}(S)}{m^2_{\tilde{K}} - S} + \frac{C^2_{\tilde{K}\pi K}(u)}{m^2_{\tilde{K}} - u}$$

$$+ 2\left[\frac{C_{\epsilon\pi\pi}{}^2(t)}{m_\epsilon^2 - t} + \frac{C_{S^*\pi\pi}{}^2(t)}{m_{S^*}{}^2 - t}\right] + (t-u)\frac{G_{K^*\pi K}{}^2(S)}{m_{K^*}{}^2 - S} + (t-S)\frac{G^2_{K^*\pi K}(u)}{m^2_{K^*} - u},$$

TABLE I.

	$a_0^{1/2}$	$a_0^{3/2}$	Δa_0	$\tau(10^{-15}\,\mathrm{s})$
Experimental[3]	0,22 ± 0,04	− 0.06	0,28 ± 0,04	3,76 ± 1,05
Experimental[3]	0.24 ± 0,02	− 0.05 ± 0.06	0,29 ± 0,08	4,25 ± 2,23
Experimental[3]	0,13	− 0,13	0,26	4,09
Nonlinear chiral model[4]			0.21	6.40
Superconducting model[5]			0.20	6.60
Virton-quark model	0,15	− 0.09	0,24	4,58

$$T^-(Stu) = -G_\Box(Stu) + G_\Box(utS) + \frac{G^2_{\tilde{K}\pi K}(S)}{m_{\tilde{K}}^2 - S} - \frac{G^2_{\tilde{K}\pi K}(u)}{m_{\tilde{K}}^2 - u}$$

$$+ 2(S-u)\frac{G^2_{\rho\pi\pi}(t)}{m_\rho^2 - t} + (t-u)\frac{G^2_{K^*\pi K}(S)}{m_{K^*}^2 - S} - (t-S)\frac{G^2_{K^*\pi K}(u)}{m_{K^*}^2 - u}$$

The calculation technique and explicit expressions for the structure integrals $G_\Box(Stu)$, $G_{VPP}(S)$, and $G_{SPP}(S)$ are given in Ref. 11. We find the scattering length from

$$a_0^{1/2} = \alpha[T^+(S_0, 0, 0) + 2T^-(S_0, 0, 0)],$$

$$a_0^{3/2} = \alpha[T^+(S_0, 0, 0) - T^-(S_0, 0, 0)],$$

where $\alpha = (1/8\pi)[1/1 + (m_K/m_\pi)]$, and $S_0 = (m_\pi + m_K)^2$. It turns out that the difference $a_0^{1/2}$ and $a_0^{3/2}$—as in a superconducting model[5]—does not depend on the contributions of the ϵ and S^* mesons, so that it is possible in principle to determine the

TABLE II.

ΔE^S_{1S}	$1.6 \cdot 10^{-3}\, E^{cul}_{1S}$	$\Delta E_{1S} = 2{,}5 \cdot 10^{-3}\, E^{cul}_{1S}$
		$E^{cul}_{1S} = -2{,}8981$ keV
ΔE^{em}_{1S}	$0.9 \cdot 10^{-3}\, E^{cul}_{1S}$	$E_{1S} = -2{,}9053$ keV
$\Delta\Psi^S_{1S}(0)$	$2{,}7 \cdot 10^{-4}\, \Psi^{cul}_{1S}(0)$	$\Delta\Psi_{1S}(0) = 4{,}2 \cdot 10^{-4}\, \Psi^{cul}_{1S}(0)$
		$\Psi^{cul}_{1S}(0) = 0{,}39035$ MeV
$\Delta\Psi^{em}_{1S}(0)$	$1{,}5 \cdot 10^{-4}\, \Psi^{cul}_{1S}(0)$	$\Psi_{1S}(0) = 0.39952$ MeV
ΔE^S_{2S-2P}	− 0.57562 эВ	
		$\Delta E_{2S-2P} = -0{,}86524$ eV
ΔE^{em}_{2S-2P}	− 0,28962 эВ	

lifetime of the πK dimesonic atom more accurately, and with fewer uncertainties of a model nature, than for the $\pi\pi$ dimesonic atom.[9] We need to emphasize that we are treating the introduction of the ϵ meson as a method for approximately describing the strong but nonresonant interaction of pions in the S wave. Since the ϵ meson has no experimental status, its mass is used as a parameter of the model.[11] Table I gives the results of calculations of the πK scattering lengths and the lifetime of the πK dimesonic atom, in comparison with values found by other approaches. Table II shows the results of corrections to the binding energy, ΔE^{S}_{1S}, the wave function of the ground state at the origin, $\Delta\Psi^{S}_{1S}(0)$, and the Lamb shift, ΔE^{S}_{2S-2P}, due to strong interactions. We see that the strong and electromagnetic corrections $[\Delta E^{\text{em}}_{1S}, \Delta\Psi^{\text{em}}_{1S}(0), \Delta E^{\text{em}}_{2S-2P}]$ from Ref. 8 are quantities on the same order of magnitude.

We wish to thank A. Karimkhodzhaev and L. L. Nememov for interest in this study and for stimulating discussions.

[1]L. L. Nemenov, Yad. Fiz. **41**, 980 (1985) [Sov. J. Nucl. Phys. **41**, 629 (1985)].
[2]S. M. Bilen'kiĭ, Yad. Fiz. **10**, 812 (1969) [Sov. J. Nucl. Phys. **10**, 469 (1970)].
[3]O. Dumbrajs *et al.*, Nucl. Phys. **B216**, 277 (1983).
[4]A. A. Bel'kov, V. N. Pervushin, and F. G. Tkebuchava, Yad. Fiz. **44**, 466 (1986) [Sov. J. Nucl. Phys. **44**, 300 (1986)].
[5]M. K. Volkov and A. A. Osipov, Report R2-85-866, Joint Institute for Nuclear Research, Dubna, 1985.
[6]S. Deser *et al.*, Phys. Rev. **96**, 774 (1954).
[7]L. D. Landau and E. M. Lifshitz, Kvantovaya mekhanika, Nauka, Moscow, 1974 (Quantum Mechanics: Non-Relativistic Theory, Pergamon, New York, 1977).
[8]A. Karimkhodzhaev and O. N. Faustov, Report R2-86-142, Joint Institute for Nuclear Research, Dubna, 1986.
[9]G. V. Afimov, M. A. Ivanov, and V. E. Lyubovitskiĭ, Yad. Fiz. **44**, 460 (1986) [Sov. J. Nucl. Phys. **44**, 296 (1986)].
[10]G. V. Efimov and M. A. Ivanov, Fiz. Elem. Chastits At. Yadra **12**, 5, 1220 (1981) [Sov. J. Part. Nucl. **12**, 1, 489 (1981)].
[11]G. V. Efimov, M. A. Ivanov, and A. M. Raportirenko, Report R2-85-198, Joint Institute for Nuclear Research, Dubna, 1985.

Translated by Dave Parsons