Transition effects in the cross section for the excitation of the *K* shell of atoms by relativistic charged particles

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The suppression of the "density effect" in the cross section for excitation of the *K* shell of the atoms near the vacuum-medium interface is examined. The calculations are in good agreement with the results of the most recent experiments with thin targets, where the suppression of the density effect is observed.

Experimental measurements of the cross section σ_k for the excitation of the *K* shell by charged particles have revealed an unexpected, at first glance, phenomenon: a partial or total suppression of the "density effect" in thin ($\sim\mu$m) targets, which should be observed at $\gamma \gtrsim \omega_K/\omega_p$ (γ is the Lorentz factor, ω_K is the *K*-shell ionization potential, ω_p is the plasma frequency, and $\hbar = c = 1$).[1-3] The physical cause of this phenomenon is, as was noted in Ref. 3, the same as that for the suppression of the density effect in the energy loss of a particle in a thin layer of material, which was predicted by Garibyan.[4] The transition effects in the local energy loss near the interface between two media were studied in Ref. 5. Using a method developed in Ref. 5, we obtained the cross section $\sigma_K(z,\gamma)$, where z is the distance travelled by the particle in the substance.

Let us assume that a particle with a charge e, which moves along the z axis from the medium $\epsilon_0(z<0)$ to the medium $\epsilon(z\geqslant 0)$, crosses the interface along the normal (ϵ is the dielectric constant). The electromagnetic field energy which is absorbed between the planes z and $z+dz$ in the material is

$$\frac{dW(z)}{dz} = \frac{1}{4\pi}\int \mathbf{E}(z,\vec{\rho},t)\,\frac{\partial(\hat{\epsilon}-1)}{\partial t}\,\mathbf{E}(z,\vec{\rho},t)\,d\vec{\rho}\,dt. \tag{1}$$

Expanding **E** in a Fourier integral, we find the following expression for the number of collisions:

$$\frac{dn(z)}{dz} = \int_0^\infty \omega\,\mathrm{Im}\,\epsilon\,\frac{dN(z)}{d\omega}\,d\omega\,; \qquad \frac{dN(z)}{d\omega} = \frac{4\pi^2}{v^2}\int\frac{d\mathbf{q}}{\omega}\,|\,\mathbf{E}(z,\mathbf{q},\omega)\,|^2 . \tag{2}$$

Equation (2) can be explained in a straightforward way in terms of the equivalent-photon method: $dN(z)/d\omega$ is the photon spectrum. The *K*-shell contribution $dn_K(z)/dz \equiv n_a\sigma_K(z)$ can be obtained if only the cross section for the photoabsorption by *K* electrons, $\sigma_K^{(\gamma)}(\omega\geqslant\omega_K)$, is taken into account in $\sigma_{\rm abs} = \omega\mathrm{Im}\epsilon/n_a$ (n_a is the density of atoms). Using the well-known field[4] $\mathbf{E}(z,\mathbf{q},\omega)$ in the transition-radiation theory, with $\omega\geqslant\omega_K$ (when $|\epsilon-1|\ll 1$) and $\gamma\gg 1\,(q^2/\omega^2\sim\gamma^{-2}\ll 1)$, we find the following expression from (2):

$$\frac{dN(z)}{d\omega} \approx \frac{e^2}{\pi\omega}\int_0^{q_0} 2q^3 dq \left| \frac{\exp(iz\omega/v)}{\Lambda} - \left(\frac{1}{\Lambda_0} - \frac{1}{\Lambda}\right)\exp\left(iz\sqrt{\epsilon\omega^2 - q^2}\right)\right|^2,$$

$$\Lambda = \frac{\omega^2}{v^2} + q^2 - \epsilon\omega^2, \quad \Lambda_0 = \frac{\omega^2}{v^2} + q^2 - \epsilon_0\omega^2. \tag{3}$$

In the limit $z \to 0$, the field $\mathbf{E}$ is the same, as can be seen from (3), as the intrinsic field $\mathbf{E}_0$ of the particle in the medium $\epsilon_0(\mathbf{E}_0 \propto 1/\Lambda_0)$. Accordingly, in the limit $z \to 0$, there will be no density effect in dW/dz, dn/dz, and σ_K (until $\gamma \lesssim \omega_K/\omega_p^{(0)}$; see Ref. 5), and these quantities will increase logarithmically with increasing γ. The width of the transition region is $z_t \cong 1/\omega_K \mathrm{Im}\epsilon(\omega_K)$. At $z \gg z_t$ the transition field $\mathbf{E}_t \propto [(1/\Lambda_0) - (1/\Lambda)]$ vanishes and the ordinary density effect is evident in the medium ϵ.

Integrating in (3) over q, we find

$$\frac{dN}{d\omega} = \frac{dN_0}{d\omega} + \frac{dN_t}{d\omega} + \frac{dN_{\mathrm{int}}}{d\omega};$$

$$\frac{dN_0}{d\omega} = -\frac{e^2}{\pi\omega\mathrm{Im}\epsilon}\mathrm{Im}\left(\tau\ln\frac{2q_0^2}{\tau\omega^2}\right);$$

$$\frac{dN_t}{d\omega} = \frac{e^2\exp(-z\omega\mathrm{Im}\epsilon)}{\pi\omega}\left\{\frac{\mathrm{Im}(\tau\ln\tau)}{\mathrm{Im}\epsilon} - \frac{\mathrm{Im}(\tau_0\ln\tau_0)}{\mathrm{Im}\epsilon_0} - 2\mathrm{Re}\frac{(\tau_0\ln\tau_0 - \tau^*\ln\tau^*)}{(\epsilon_0 - \epsilon^*)}\right\};$$

$$\frac{dN_{\mathrm{int}}}{d\omega} = \frac{e^2}{\pi\omega}2\mathrm{Re}\left\{\exp\left(-\frac{iz\omega\tau}{2}\right)\left[\frac{f(\tau_0) - f(\tau^*)}{\epsilon_0 - \epsilon^*} - \frac{\mathrm{Im}f(\tau)}{\mathrm{Im}\epsilon}\right]\right\}, \tag{4}$$

where $\tau = v^{-2} - \epsilon$, $f(\tau) = \tau\exp(iz\omega\tau/2)\mathrm{Ei}(-iz\omega\tau/2)$, v is the particle velocity, and Ei is an exponential integral. The contribution from the "near" collisions ($q \gg q_0$) must be added to $dN_0/d\omega$ in (4) (the other terms converge as $q \to \infty$). We will make this addition by using the method which takes into account the spatial dispersion in $\epsilon(\omega,q)$ and which is used effectively in the calculations of the ionization effects (the photoabsorption ionization model).[6,7] In our case this method leads to the substitution $q_0^2 \to 2m\omega$ and to the addition of the term

$$\frac{dN_0^{(1)}}{d\omega} = \frac{e^2}{\pi\omega^3\mathrm{Im}\epsilon}\int_{\omega_K}^{\omega}\omega'\,\mathrm{Im}\,\epsilon(\omega')d\omega'. \tag{5}$$

The sum (5) and $dN_0/d\omega$ in (4) yield an equivalent-photon spectrum in a homogeneous medium, and at $\mathrm{Im}\epsilon \approx 0$ the factor of the exponential function in $dN_t/d\omega$ gives the spectrum of the transition quanta, in which absorption and Čerenkov radiation are taken into account. If $\epsilon \equiv 1$, this spectrum will coincide with the result of Ref. 8. At $z \lesssim z_t$ individual contributions to $dN/d\omega$ do not have a profound physical meaning.

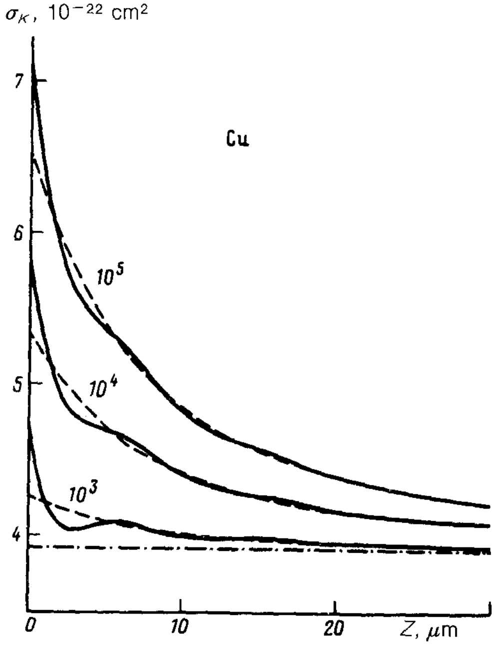

FIG. 1. The cross section σ_K for Cu plotted as a function of the depth z in the target for different values of γ. Dot-dashed curve—σ_0 in the limit $\gamma \to \infty$; dashed curves—$\sigma_0 + \sigma_t$; solid curves—$\sigma_0 + \sigma_t + \sigma_{\rm int}$. The labels on the curves are the values of the Lorentz factor γ.

The cross section for the excitation of the K shell

$$\sigma_K(z, \gamma) = \sigma_0(\gamma) + \sigma_t(z, \gamma) + \sigma_{\rm int}(z, \gamma)$$

can be calculated by using the photoabsorption cross sections $\sigma_K^{(\gamma)}(\omega)$ and $\sigma_{\rm abs}(\omega)$ from Ref. 9. The complex function $\epsilon(\omega)$ was found from the Kramers-Kronig rela-

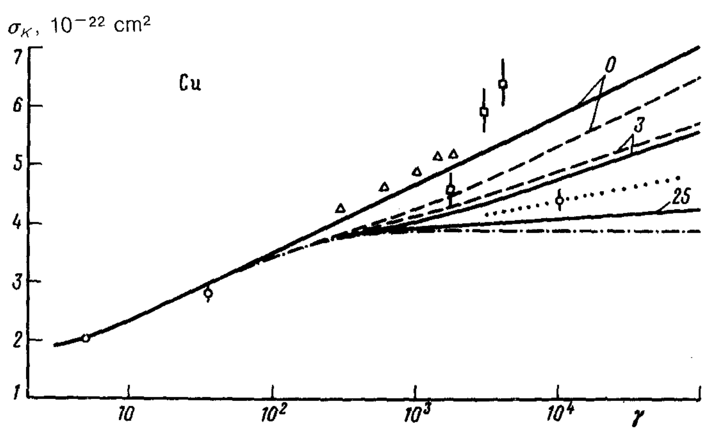

FIG. 2. The cross section σ_K for Cu plotted as a function of γ for various values of z. Dot-dashed curve—$\sigma_0(\gamma)$; dashed curves—$\sigma_0 + \sigma_t$; solid curves—$\sigma_0 + \sigma_t + \sigma_{\rm int}$. The labels on the curves are the values of z in microns. Dotted curve—calculation of the relative increase of the cross section in the interval $5 \leqslant \gamma \leqslant 5 \times 10^4$ for the conditions under which the experiments of Ref. 3 were carried out, with allowance for the absorption of K_α photons in the target, $\lambda_\alpha = 22.3$ μm. The experimental data: Δ—Ref. 1; □—Ref. 2; ○—Ref. 3.

tions with respect to $\mathrm{Im}\epsilon(\omega)$. Spectra (4) and (5) were calculated under the assumption that $\epsilon_0 \equiv 1$. The integration over ω in (2) was carried out numerically. As can be seen in Figs. 1 and 2, the interference term in (4) plays largely a minor role. At $z \ll z_t$, however, this term must be taken into account in order to obtain a physically correct result: In this approximation the equivalent-photon spectrum should be continuous at the boundary. In the relativistic case, where the particle blindly collides with the target, the relations which we obtained can justifiably be used to calculate $\sigma_K(z,\gamma)$ in layers of finite thickness.

A good agreement between the calculated values of $\sigma_K(z)$ and the experimental results graphically illustrates the suppression of the density effect near the interface between two media, a topic long discussed in the transition-radiation theory.

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