Ionization self-channeling of extremely intense electromagnetic waves in a plasma

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Ionization self-channeling of radiation in a plasma, which is accompanied by a high concentration of the electromagnetic field energy, is detected.

The plasma dynamics in extremely intense electromagnetic fields, in which the oscillator energy of electrons is higher than the ionization energy of molecules,[1,2] $\mathscr{E}_{\sim} \gg \mathscr{E}_i$, has recently been the subject of considerable attention, principally because of the rapid increase in the intensity of modern radiation sources. The laws governing the formation of a plasma and its effect on the propagation of electromagnetic waves in a given energy range differ radically from the well-known effects[3,4] for moderate-intensity fields when $\mathscr{E}_{\sim} \lesssim \mathscr{E}_i$. One of the more interesting processes, which characterizes the plasma in extremely intense fields as a medium with unusual nonlinear properties, is the ionization self-channeling of radiation which accounts for the high concentration of electromagnetic energy.

The ionization self-effect can be explained qualitatively in terms of the electron-impact-induced decrease of the ionization frequency ν_i with increasing electric-field amplitude, which is characteristic for extremely strong fields. In the course of propagation of a packet of electromagnetic waves, the plasma density increases slower along its axis than at its periphery during the ionization, resulting in the formation of a nonuniform dielectric-constant profile, which accounts by virtue of refraction, for a further field concentration in the central part of the packet.

To construct a quantitative model, we will use a simple constitutive equation for electron density n in a weakly ionized plasma

$$\partial n/\partial t = \nu_i n \, . \tag{1}$$

This equation is valid for the spatial scales of the process which are substantially greater than the length of electron-impact-induced ionization of the gas molecules in extremely intense fields if the local loss of charged particles is ignored. For the ionization frequency, which in this case is the local function of the amplitude A of the electric field, we use a power-law approximation[1)]

$$\nu_i(A) = \nu_0 A_0/A. \tag{2}$$

Assuming that the electron density is small in comparison with the critical density ($n \ll n_c = m\omega^2/4\pi e^2$), w can describe the complex amplitude E of the rf electric field, $\mathbf{E} = \mathbf{y}^0 E(x,z,t) e^{i\omega t - ikz} + \text{c.c.}$, by means of a parabolic equation[2)]

$$-\frac{2i}{\omega}\frac{\partial E}{\partial t}-\frac{2i}{k}\frac{\partial E}{\partial z}+\frac{1}{k^2}\frac{\partial^2 E}{\partial x^2}-\frac{n}{n_c}E=0. \tag{3}$$

Equations (1)–(3) can easily be analyzed in the new dimensionless variables $t_n=\delta\omega t$, $z_n=\delta kz$, $x_n=\sqrt{2\delta}kx$, $E_n=E/A_0$, $n_n=n/2\delta n_c$, $\delta=\nu_0/\omega$, $k=\omega/c$, and $A_n=|E_n|=A/A_0$.

In the approximation of the quasiplane phase front, whose radius of curvature is $R\gg kL^2$ (L is the scale width of the wave packet), the equations formulated above have a broad range of self-similar solutions which describe the waveguide channels that contract adiabatically over time[3]:

$$A=[t+f(z)]F(\xi),\quad n=[t+f(z)]^4\Phi(\xi)/P^2,\quad \xi=[t+f(z)]^2x/P, \tag{4}$$

where $f(z)$ is an arbitrary function, and $P=\text{const}$ is a parameter which is proportional to the power transmitted through the channel. The transverse structure of these channels is determined by a system of equations in ordinary derivatives,

$$\frac{d^2F}{d\xi^2}+F-\Phi F=0,\qquad 2\xi\frac{d\Phi}{d\xi}+4\Phi-\Phi/F=0. \tag{5}$$

A symmetric solution of (5) with respect to ξ, which is shown in Fig. 1 at $\Phi(0)\ll 1$ is given by

$$F=\frac{1}{4}\cos\xi,\qquad \Phi=\Phi(0)\exp\left[\sum_{k=1}^{\infty}\frac{|E_{2k}|\xi^{2k}}{k(2k)!}\right],$$

In this expression E_{2k} is a sequence of Euler's numbers. At the periphery of the channel, where $A\to 0$, n has a characteristic feature: Here approximation (2) no longer holds, and the plasma density reaches a steady-state value under actual conditions. The initial approximation of the quasiplane phase front is valid if the initial width of the wave packet is small: $P^2|f'+1|\ll f^5$. At $f(z)=-z$ expressions (4) describe the exact solution of Eqs. (1)–(3). The phase front in this case is planar.

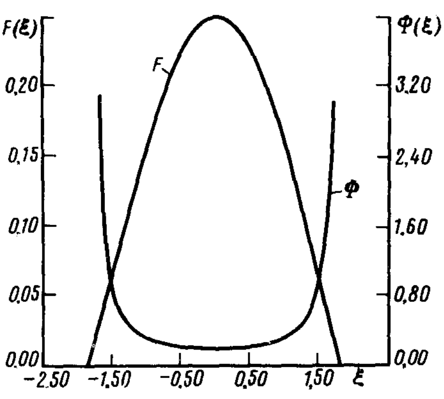

FIG. 1.

Analysis of the dynamics of the self-effect of the wave packets on the basis of the nonaberrational approximation[6] has shown that the solutions found above are a stable manifold. Writing the transverse field and plasma distributions near the packet axis as a series

$$A = \frac{1}{4}[P/a(z,t)]^{1/2}\left(1 - \frac{x^2}{a^2} + \dots\right), \qquad n = U(z,t)\left[1 + \frac{x^2}{b^2(z,t)} + \dots\right], \qquad (6)$$

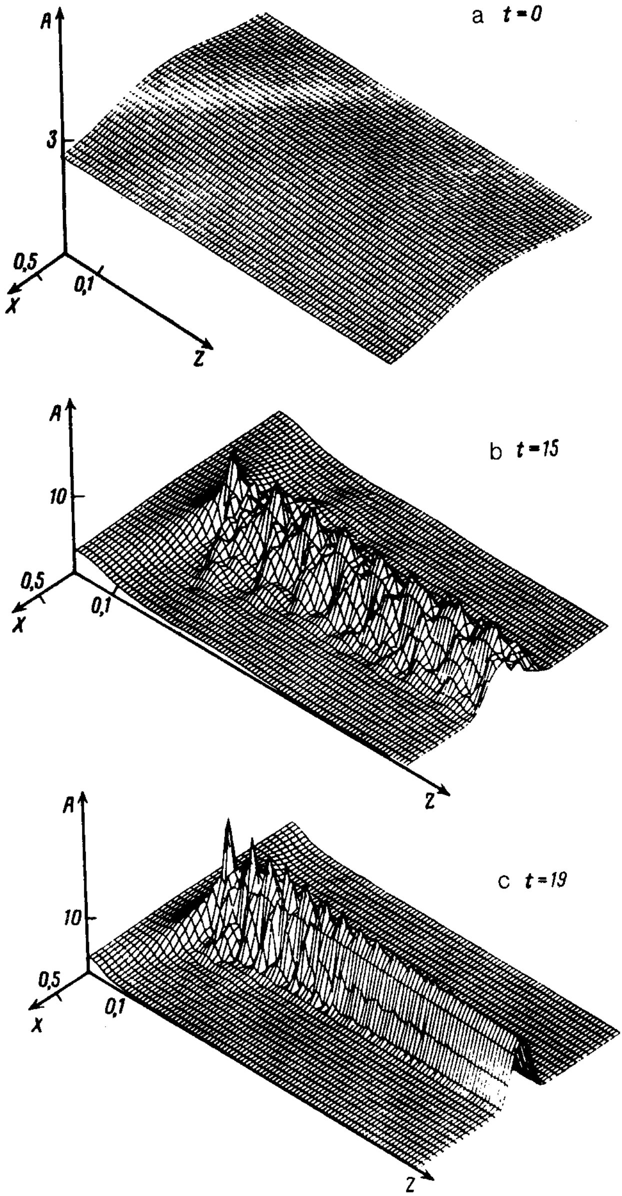

FIG. 2.

we can use the equations for the new functions

$$\left(\frac{\partial}{\partial t}+\frac{\partial}{\partial z}\right)^2 a=\frac{16}{a^3}-\frac{4aU}{b^2},\quad \frac{\partial U}{\partial t}=\frac{4Ua^{1/2}}{P^{1/2}},\quad \frac{\partial b}{\partial t}=-\frac{2b^3}{P^{1/2}a^{3/2}}. \tag{7}$$

Analysis of Eqs. (7) shows that the solutions of these equations in time reach the adiabatic compression stage ($a^4 \simeq 4b^2/U$), which corresponds to the approximation of the quasiplane phase front. It is asymptotically valid to write

$$a=b=\frac{P}{t^2}\left(1+\frac{\alpha}{tz^{1/4}}\right),\quad U=\frac{4t^4}{P^2}\left(1-\frac{2\alpha}{tz^{1/4}}\right),\quad \alpha=\text{const}.$$

A numerical experiment was carried out to determine whether a radiation produced by a steady source at the boundary with the medium can be captured into a contracting uniform channel which was calculated above. In the solution of Eqs. (1)–(3), the functional dependence $\nu_i(A)$ was chosen for $A^2>3$ in the form $\nu_i=1/A$ and for $A^2<3$ in the form $\nu_i=1/\sqrt{3}$. Figure 2 shows the initial quasiuniform distribution (in the absence of a plasma) of the field amplitude in space and a structure that forms during the self-effect ("b" refers to that structure which is formed at the stage of self-similar compression).

At the final stage of self-channeling, model (1)–(3), which was examined above, breaks down, since the beam width either is equal to the wavelength (at this time the electron density reaches the critical value) or it is equal to the ionization length. In the first case, a further increase of the ionization may cause the incident electromagnetic wave to be reflected. If, on the other hand, the ionization length is greater than the radiation wavelength (this situation can occur in a sufficiently rarefied gas), then the collisionless nonlinearity mechanisms will come into play as the channel is contracted. In this case we can expect that the combined effect of ionization and striction in the ionized region of the gas will give rise to the formation of a narrow, plasma-free channel which will trap an appreciable part of the electromagnetic energy.

1) Determination of the exact functional dependence $\nu_i(A)$ is a complex independent problem which can be solved only in several limiting cases. Numerical analysis of this functional dependence on the basis of the kinetic equation were carried out in Refs. 1 and 5.

2) The dissipative effects in Eq. (3) can be ignored, since the transport cross section for the electron collisions is small compared with the ionization cross section.

3) The solutions found here can easily be generalized to the case of an axisymmetric wave packet and to other exponents of the functional dependence $\nu_i(A)$. In particular, in approximation (2) the channel width decreases as $1/t$ in the case of an axisymmetric channel, while the plasma density increases as t^2.

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