Vortices on the world sheet of a string; critical dynamics

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Vortices on the world sheet of a string which is moving in a multiply connected space lead to a continuous dependence of the critical indices, in particular, the critical dimensionality, on the parameters of cycles which cannot be contracted. There exist limiting values of the latter quantities because of a phase transition associated with a loss of conformal symmetry. Numerous applications are discussed.

Let us examine a string in the formalism of summations over surfaces,[1] in which string dynamics is determined by a two-dimensional conformal theory on a world sheet[2] which is propagating in a multiply connected space M. For simplicity, we will speak exclusively in terms of boson strings here. The generalization to the case of superstrings requires some slight modifications. It will be published separately, along with detailed explanations of the assertions made below. A nontrivial $\pi_1(M)$ arises in a broad range of problems, e.g., (a) compactification onto tori generated by lattices, in particular, onto the maximal tori of Lie groups generated by root lattices in the heterotic-string model,[3] where

$$\pi_1(M) = \underbrace{Z \times \dots \times Z}_{r},$$

where r is the rank of the group; (b) compactification onto multiply connected Calabi-Yao manifolds or orbifolds,[4] where $\pi_1(M)$ is a finite group, possibly non-Abelian; and (c) strings at finite temperatures,[5] with a space which is periodic in an imaginary time with the period T^{-1}, with the topology $M = R^{D-1} \times S^1$, and $\pi_1(M) = Z$. The radius R is related to the temperature by $R = (2\pi T)^{-1}$. This example may also be thought of as an extremely simple compactification onto S^1.

In addition, a nontrivial π_1 arises in a description of metrics of the black-hole type, with identified right and left worlds, against the background of cosmological strings, and so forth. We will begin our analysis of vortices with case (c).

1. The action of the string is

$$S = \frac{1}{4\pi\alpha'} \int d^2z \sqrt{g} g^{ab}(z) \partial_a x^\mu \partial_b x^\nu g_{\mu\nu}(x) ,$$

where $g_{ab}(z)$ and $g_{\mu\nu}(x)$ are the metrics on the world sheet and in a nested space. In the critical dimensionality we can avoid $g_{ab}(z)$ and obtain the action of a two-dimensional conformal theory of the fields x^μ alone. Corresponding to the compact dimension in which we are interested is the action

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$$S = \frac{1}{4\pi\alpha'} \int d^2z\, \partial_a x \partial_a x = (R^2/4\pi\alpha') \int d^2 z \partial_a \varphi \partial_a \varphi ; \quad \begin{matrix} \varphi \in [0, 2\pi] \\ a = 1,2 \end{matrix} , \tag{1}$$

which describes a continuous limit of the XY model or of a planar magnet. Berezinskiĭ[6] undertook a detailed study of this model, which was continued by Kosterlitz and Thouless[7] (the BKT model). Two phases exist: a low-temperature phase with conformal invariance and a high-temperature phase with exponentially decreasing correlation functions. The string familiar to us is described by the first phase alone. The reason for the BKT phase transition is the creation of vortices: nonperturbative fluctuations with a logarithmically large action. The appearance of such fluctuations is not an artifact of the regularization (on a lattice in Refs. 6 and 7). In particular, the naive Pauli-Willars regularization does not work, since the mass term breaks the symmetry $\varphi \Rightarrow \varphi + 2\pi n$, while a potential of the type $M^2 \cos\varphi$ leads to an infinite self-effect in the limit $M \to \infty$. That result is a manifestation of the same vortex effects.

In the nonconformal phase the vortices form a two-dimensional Coulomb plasma in which Debye screening leads to a loss of the conformal properties, while in the conformal phase the vortices and antivortices are bound into dipoles. The field and the action of a vortex with a charge q [mapping class $\pi_1(S^1)$] are

$$\varphi(z) = -\frac{i}{2} q \ln[(z - z_0)/(\bar{z} - \bar{z}_0)], \qquad S_q = \pi\beta q^2 \ln(A/a^2) , \tag{2}$$

where $z = x + iy$, $\beta = R^2/4\pi\alpha'$, A is the area of the world sheet, and a is the spacing in the lattice—the ultraviolet cutoff. A phase transition occurs when the creation of a vortex is thermodynamically favorable. Its free energy is zero, and since the entropy of the vortex is obviously $\ln(A/a^2)$, we can write

$$F_q = (\pi\beta q^2 - 1)\ln(A/a^2), \quad |q| = 1, \quad \beta_c = \pi^{-1}, \quad R_c = 2\sqrt{\alpha'}.$$

It is for specifically this R_c that the soliton sector contains additional massless states, while at $R < R_c$ the theory would have some additional techyons, so that we can see the relationship between the loss of conformal properties on the world sheet and the appearance of additional tachyons. The mass spectrum in string theory is (L is the soliton number, and m is the index of the momentum p_φ):

$$\frac{\alpha' M^2}{2} = 2(N-1) + \frac{\alpha' m^2}{2R^2} + \frac{L^2 R^2}{2\alpha'} + mL; \quad m, L = 0, \pm 1, \ldots \quad . \tag{3}$$

If we now switch from R_c to a critical temperature $T_c = (2\pi R_c)^{-1}$, we find $T_c = (4\pi)^{-1}(\alpha')^{-1/2}$. Comparing it with the limiting temperature in string theory, i.e., the Hagedorn temperature T_H, which would be given in the case of a boson string by[5] $T_H = (2\pi)^{-1}[(D-2)\alpha'/6]^{-1/2}$, where D is the dimensionality of the space-time, we find

$$D = 26, \quad T_c = T_H, \quad D < 26, \quad T_c < T_H, \quad D > 26, \quad T_c > T_H \ !$$

The actual reason for the divergence in the Gibbs ensemble of strings is a BKT phase transition on a world sheet and the loss of conformal symmetry!

In the conformal dipole phase, corrections to the correlation functions which are nonperturbative in α' arise, and as a result there are corrections to all the critical parameters of the two-dimensional conformal theory. In the one-dipole approximation we have

$$\langle \varphi(z)\, \varphi(0) \rangle = - (8\pi\beta)^{-1} \left(1 + 8\pi^2 \frac{\beta}{\beta - \beta_c} \exp(-\mu\beta)\right) \ln z^2 \,, \tag{4}$$

where μ is the minimal energy of the dipole—the chemical potential. From (4) we find corrections to the coefficients in the operator expansions which determine, for example, the mass spectrum of the string and the critical dimensionality. In the former case, we deal with the product of the vertex operator $V(z)$ and the two-dimensional energy-momentum tensor $T(z)$:

$$V(z)T(z') = \Delta_V (z - z')^{-2} V(z) + (z - z')^{-1} \partial_z V + \ldots \,.$$

From conformal symmetry we have $\Delta_V = 2$; finding Δ_V from (4), we then find the spectrum

$$\frac{M^2\alpha'}{2} = 2(N-1) + \frac{m^2\alpha'}{2R^2} \left(1 + 8\pi^2 \frac{R^2}{R^2 - 4\alpha'} \exp(-\mu R^2/4\pi\alpha')\right). \tag{3'}$$

From (3') we find a renormalization of R_c due to the dipole-dipole interaction; this renormalization has been found previously by renormalization-group methods.[7,8] The corrections to the central charge of the Virasoro algebra are found from

$$T(z)\, T(z') = \frac{c}{2} (z - z')^{-4} + \ldots \,.$$

We find

$$c_\varphi = \left(1 + 8\pi^2 \frac{\beta}{\beta - \beta_c} \exp(-\mu\beta)\right)^2 ; \qquad \sum_{i=1}^{D} c_i = 26. \tag{5}$$

This result tells us that the critical dimensionality deviates from its standard value in the presence of a cycle (or cycles) which cannot be contracted! It thus becomes possible to construct a theory in other critical dimensionalities. Corrections to the equations of motion of massless fields (the graviton, the dilaton, etc.) also arise; in particular, a cosmological term which depends on the parameters, e.g., T, is generated:

$$\Lambda = (8/3)\pi^2 (\alpha')^{-1} \exp(-\mu/16\pi^3\alpha' T^2).$$

2. We turn now to lattices. The interesting action in this case is

$$S = \beta \int d^2 z\, a_{ij}\, \partial_a \varphi^i \partial_a \varphi^j \,; \qquad i, j = 1, \ldots r$$

where a_{ij} is the matrix of the lattice. This model generalizes the XY model to the case of several "colors." It again has vortices, but in this case the colors interact. The correlation function in this model is of the form $\langle \varphi^i(z)\varphi^j(0)\rangle = -(8\pi\beta)^{-1}a_{ij}^{-1}\ln z^2$. In the one-dipole approximation the correction is proportional to δ_{ij}, while in the two-dipole approximation it is proportional to a_{ij} (to the correlation energy of dipoles with the colors i and j). The higher-order corrections reduce to either δ_{ij} or powers of a_{ij}:

$$\langle \varphi^i(z)\varphi^j(0)\rangle = -(8\pi\beta)^{-1}(a_{ij}^{-1} + c_1\delta_{ij} + c_2 a_{ij} + \dots)\ln z^2 . \tag{6}$$

It follows from (6) that only self-dual lattices with $a_{ij} = a_{ij}^{-1}$ retain their structure in the face of dipole renormalizations, and it is specifically these lattices which are used in the model of a heterotic string! If we are studying a lattice with a Lorentzian signature, then the components which have the incorrect sign of the action correspond to an antiferromagnet. Again in this case, nonperturbative effects are important.

3. We conclude with a few words about compactification. In addition to the possibility of other dimensionalities, which we have already mentioned [see (5)], there is a mechanism for the generation of condensates of external fields with decreasing $R < R_c$. This situation gives rise to the formation of a vortex phase described by the action[10]

$$S = \int d^2z\,(\beta\partial_a\chi\partial_a\chi + \lambda\cos\beta\chi).$$

If we now consider a sine-Gordon model in the external field of a tachyon, it can contract the potential and restore conformal symmetry. In the supercase, a Yang-Mills field will precipitate in a condensate, since the sine-Gordon superpotential is $\bar{\psi}\psi\cos\beta\chi$. This is precisely the form of the required part of the vertex operator of a Yang-Mills field. This mechanism may explain the subsequent breaking of gauge symmetry in the superstring model.[9,4]

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