Strong interaction in a quark-gluon plasma above the critical temperature

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The nonperturbative interaction between quarks and gluons is also strong above the critical temperature. It gives rise to a characteristic spectrum.

Naive considerations would lead us to expect that asymptotic freedom should give rise to a free gas of quarks and gluons in the limit of high temperatures T (Ref. 1). Indeed, calculations of global thermodynamic quantities, e.g., the free energy and the entropy, by the Monte Carlo method support this expectation.[2] In the limit $T \to \infty$,

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on the other hand, the QCD Lagrangian becomes the effective Lagrangian of a $3D$ Euclidean QCD,[3] in which there is confinement, and the interaction is strong.

In this letter we use the formalism of the method of vacuum correlation functions.[4-6] We show that deconfinement at $T>T_c$ in a $3+1$ QCD is indeed accompanied by confinement in three dimensions (i.e., spatial Wilson loops which obey the area law). We also determine the $(q\bar{q})$ interaction at $T>T_c$, and we find the corresponding spectrum.

We would first like to discuss Wilson loops in a (4,1) plane and also in spatial planes, e.g., (1,2). We use a cluster expansion.[4] Retaining only the lowest-order terms quadratic in the fields, we find the following result for large areas R, $R_4 \gg t_g$, where t_g is the vacuum correlation length,[6] which determines the decay law of the correlation function $\langle F(x)F(0)\rangle$:

$$< W_{\mu\nu} >= \exp(-\sigma_{\mu\nu} S - C) . \tag{1}$$

Here S is the area inside the loop, and C incorporates terms of the perimeter type and Coulomb corrections,

$$\sigma_{14} \equiv \sigma_E = \int d^2 z D^E(z), \sigma_{ik} \equiv \sigma_B = \int d^2 z D^B(z). \tag{2}$$

Here we have introduced some nonperturbative correlation functions for the color electric and color magnetic fields:

$$< E_i(z)E_j(0) >= \delta_{ij} D^E(z) + \frac{1}{2}\frac{\partial}{\partial z_4}(z_4 D_1^E)\delta_{ij} + \frac{1}{2}\frac{\partial}{\partial z_i}(z_j D_1^E), \tag{3}$$

$$< B_i(z)B_j(0) >= \delta_{ij} D^B(z) + \frac{1}{2}\delta_{ij}\frac{\partial}{\partial z_K}(z_K D_1^B) - \frac{1}{4}\frac{\partial}{\partial z_j}(z_i D_1^B). \tag{4}$$

At $T=0$, we have the relationships[4,6] $D^E \equiv D^B$ and $D_1^E \equiv D_1^B$, and these quantities depend on $\sqrt{\bar{z}^2 + z_4^2}$.

At $T>0$, all the D depend on z_4 in a cyclic manner with a period $\beta = 1/T$ and also on $|\bar{z}|$.

At $T \geqslant T_c$, a deconfinement transition occurs; i.e., we have $\sigma_{14}=0$. The reason is that the potential between the statistical charges which corresponds to (1) is $V=\sigma_E R$. By virtue of (2), this circumstance means that we have $D_E \equiv 0$ at $T \geqslant T_c$.

For spatial Wilson loops, on the other hand, calculations and analytic estimates[7] show that the area law remains in force at $T>T_c$; i.e., we have $\sigma_{ik} \neq 0$ and $D_B \neq 0$ at $T>T_c$.

These ideas are supported by lattice calculations[8] of the gluon condensate

$$< E_{\mu\nu} F_{\mu\nu} >\sim D^E(0) + D^B(0) + D_1^E(0) + D_1^B(0), \tag{5}$$

which have shown that this quantity does not change abruptly at $T=T_c$. It remains on the same order of magnitude at $T>T_c$. Consequently, all the facts require that we have $D_E=0$ and $D_B \neq 0$ at $T>T_c$. These facts do not contradict the conditions

$D_1^E \neq 0$, $D_1^B \neq 0$. Below we work from this understanding to derive the $q\bar{q}$ interaction. For simplicity we restrict the analysis to the nonrelativistic case, and we assume[6] that we can write the interaction potential in the form

$$V(r) = V^E(r) + V^B(r), \tag{6}$$

where the term V^E, due to the electric correlation functions, is

$$V^E(r) = \epsilon(r) - \frac{4}{3}\alpha_s \frac{e^{-m_{el} r}}{r}, \tag{7}$$

where

$$\epsilon(r) = \int_0^r \lambda d\lambda \int_0^\beta d\nu D_1^E(\lambda, \nu). \tag{8}$$

The quantity m_{el} arises because of screening in the plasma; in lowest order, it is given by[9]

$$m_{el}^2 = g^2(T)(1 + N_f/6)T^2. \tag{9}$$

The potential V^B incorporates all the spin-dependent terms, including the hyperfine and tensor interactions. Explicit expressions for those interactions are given in Ref. 6 [see Eqs. (71) and (72) of that paper, where the replacements $\beta D \to D^B$, $\beta D_1 \to D_1^B$ must be made]. All these terms, however, except the spin-orbit forces, have a radius $\sim t_g$, which is small by assumption. We restrict the present discussion to the spin-orbit forces. We thus write

$$V^B(r) = -\frac{\mathbf{LS}}{m^2 r}\sigma_B, \quad r \gg t_g. \tag{10}$$

We turn now to the spectrum corresponding to $V(r)$, putting aside for the time being the need to deal with an ensemble at thermal equilibrium. The quantity D_1^E in Eq. (8) can be estimated from the calculations of Ref. 10, where D_1 was positive in one version.[11] Using an exponential parametrization for D_1, we can find D_1^E with parameter values which satisfy the condition for the appearance of levels. In this case $\epsilon(r)$ is a well with a behavior $\epsilon(r) \sim r^2$ as $r \to 0$ and $\epsilon(r) \to \text{const} > 0$ as $r \to \infty$. The quark and the antiquark are thus bound but there exists a threshold $\epsilon(\infty)$ above which the quarks fly apart, each acquiring a nonperturbative mass increment $\Delta m = \frac{1}{2}\epsilon(\infty)$. In addition, interaction (10) is important at a sufficiently high temperature, where Coulomb screening in (7) can be ignored, because of the increase in m_{el} and the decrease in $g^2(T)$. The spectrum corresponding to (10) has an unusual property: It arises only in the cases $\mathbf{LS} > 0$. In other words, for $J = L + 1$ it also has an accumulation point at

$$M = 2m - \frac{\sigma_B^2}{4m^3} \frac{L^2}{(n_r + L + 1)^2} \to 2m - \frac{\sigma_B^2}{4m^3}. \tag{11}$$

Turning now to an ensemble of quarks and gluons at $T > T_2$, we note that the spectrum in this case is related to so-called linear response functions and may be manifested in hadron emission cross sections.[12] The states of the spectrum can also be associated with so-called statistical screening lengths which were calculated by the Monte Carlo method in Ref. 13. Hadron signals were found even at $T > T_c$, and the states of mesons and baryons with opposite parities had approximately equal masses.[13] Qualitatively the same picture can be associated with our own spectrum. As we mentioned above, at $D_1^E > 0$ the potential $\epsilon(r)$ may give rise to a spectrum of bound states until the temperature T reaches a level on the order of the quark dissociation threshold $\epsilon(\infty)$. There might accordingly be a soft transition from the hadron spectrum at $T < T_c$ to a modified hadron spectrum at $\epsilon(\infty) > T > T_c$, in which free quarks do not yet appear.

In the limit $T \gg \epsilon(\infty)$, the quarks become nearly free: All interactions are small in comparison with the temperature. In this case, however, the Wilson loops in the (1,2), (1,3), and (2,3) planes nevertheless obey the area law.

We have thus shown that it is a simple matter to explain the compatibility of deconfinement in the time direction with confinement along the three spatial directions in terms of the correlation functions D and D_1. We have also shown that it is possible to derive the strong interaction between quarks, which gives rise to a corresponding spectrum in the transition region $\epsilon(\infty) > T > T_c$. A shortcoming of our analysis is the uncertainty regarding the functions D_1^E and D_1^B, which cannot be calculated from the theory and which cannot yet be reconstructed unambiguously from quarkonium spectra, even at absolute zero. It would be very important to calculate all the correlation functions D and D_1 on a lattice for both $T < T_c$ and $T > T_c$. The dynamics of the quarks can then be reconstructed unambiguously from Eqs. (8) and (10).

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