Strong-interaction corrections to the Abelian axial anomaly

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The corrections of leading order in the strong-interaction constant $\bar{\alpha}_s$ to the expression for the Abelian axial anomaly are calculated. The result is $\partial^\alpha J^5_\alpha = (\alpha/4\pi)F\tilde{F}[1 + C_F(\bar{\alpha}_s/\pi)]$ when the normalization of the axial current is $\gamma^\alpha(\langle q|J^5_\alpha|q\rangle^{\text{one loop}} - \langle q|J^5_\alpha|q\rangle^{\text{tree}}) = 0$.

Anomalies play an important role in the theory of elementary particles today (see Ref. 1, for example, as a review). The Abelian axial anomaly, which was discovered and first studied more than 20 years ago,[2] leads to a predicted decay width for the neutral pion which is in excellent agreement with experiment.[3] Perturbation-theory corrections to the expressions for the anomalies have recently been discussed extensively in the literature (see, for example, Ref. 4 and the bibliography there). The Adler–Bardeen theorem[5] guarantees only that there will be no anomalies in higher-order loops if none appear in the simple one-loop diagrams. One way to discuss the anomalies which do exist is in terms of operator relations such as

$$\partial^\alpha J^5_\alpha = \frac{\alpha}{4\pi}F\tilde{F}, \tag{1}$$

where $J^5_\alpha = (\bar{u}\gamma_\alpha\gamma_5 u - \bar{d}\gamma_\alpha\gamma_5 d)/2$ is the third component of the Abelian axial isotriplet, and $F_{\mu\nu}$ is the electromagnetic stress tensor. Such a discussion is meaningful only if the constituent operators j^5_μ and $F\tilde{F}$, which appear in (1) in the corresponding order of the perturbation-theory expansion, are rigidly defined.

In this letter we are reporting calculations of the corrections for strong interactions to the expression for the Abelian axial anomaly.

A three-point Green's function satisfying the Bose-symmetry conditions in the kinematics $k_1^2 = k_2^2$, where k_1 and k_2 are the photon momenta, can be expanded in three independent tensor structures. We choose these structures in the form

$$\begin{aligned} T^{\mu\nu\alpha}(k_1,k_2) &= \int \langle 0|TJ^\mu(x)J^\nu(y)J^{5\alpha}(0)|0\rangle e^{ik_1x+ik_2y}dxdy \\ &= (q^\mu\epsilon^{\nu\alpha\rho\tau}p_\rho q_\tau + q^\nu\epsilon^{\mu\alpha\rho\tau}p_\rho q_\tau)F_1(q^2,p^2,pq) \\ &+ (p^\mu\epsilon^{\nu\alpha\rho\tau}p_\rho q_\tau - p^\nu\epsilon^{\mu\alpha\rho\tau}p_\rho q_\tau)F_2(q^2,p^2,pq) \\ &+ \epsilon^{\mu\nu\alpha\tau}q_\tau F_3(q^2,p^2,pq), \end{aligned} \tag{2}$$

where $k_1 = p + q$, $k_2 = p - q$, and J_μ is the electromagnetic current.

From conservation of the vector current, $\partial^\mu J_\mu = 0$, we find equations for the invariant amplitudes F_i:

$$k_{1\mu} T^{\mu\nu\alpha}(k_1, k_2) = 0 = \epsilon^{\nu\alpha\rho\tau} p_\rho q_\tau (q^2 F_1 + p^2 F_2 + F_3), \tag{3a}$$

$$k_{2\mu} T^{\mu\nu\alpha}(k_1, k_2) = 0 = -\epsilon^{\mu\alpha\rho\tau} p_\rho q_\tau (q^2 F_1 + p^2 F_2 + F_3). \tag{3b}$$

Taking the divergence of the axial current, we find the equation

$$p_\alpha T^{\mu\nu\alpha}(k_1, k_2) = \epsilon^{\mu\nu\rho\tau} p_\rho q_\tau F_3. \tag{4}$$

We take the limit $q^2 \to \infty$, with p^2 fixed, in (3):

$$\lim_{q^2 \to \infty} q^2 F_1(q^2, p^2, pq) = -F_3. \tag{5}$$

Relation (5) plays a key role in the entire analysis below. It is easy to see that the amplitude F_1 is finite in the leading order and does not require regularization. Putting $T_{\mu\nu\alpha}$ in symmetric form with respect to μ and ν and thereby singling out the invariant function F_1 from the entire amplitude $T_{\mu\nu\alpha}$, we find, after an elementary integration,

$$\lim_{q^2 \to \infty} q^2 F_1 = \pi^{-2}.$$

Using (4) and (5), we then find the correct expression for the anomaly in (1).

We turn now to a calculation of the corrections to three-point function (2) for strong interactions. The sum of all the two-loop diagrams is finite by virtue of the Ward identities for the vector current and for the divergent contributions of the axial current, but there is some arbitrariness in the determination of the finite part of the axial current in this order in the strong-coupling constant $\bar{\alpha}_s$. The reason for this arbitrariness is that there is no symmetry requirement in the specification of this current, in contrast with the case of the vector current. This arbitrariness also determines the normalization of the axial current, whose anomalous divergence we are calculating. In practice, it is convenient to use the Landau gauge and an intermediate dimensional regularization in order to calculate the two-loop integrals. There is no need to redefine the γ_5 matrix. The corrections to the propagator for the massless fermion are zero in the Landau gauge. We will discuss the calculation of the vertices in slightly more detail. For example, the contribution to the invariant amplitude F_1 from the diagram associated with the correction to the axial vertex is finite in the Landau gauge; again, we do not need to regularize it. Making use of the cyclic nature of the trace, we write the expression for this diagram in the form $\mathrm{tr}[\gamma_5 \Gamma_{\mu\nu\alpha}(p,q)]$, where

$$\Gamma_{\mu\nu\alpha}(p,q) = \int dk dl S(k-p)\gamma^\sigma S(l-p)\gamma_\nu S(l-q)$$
$$\times \gamma_\mu S(l+p)\gamma^\tau S(k+p)\gamma_\alpha D_{\sigma\tau}(k-l) + (\mu \to \nu).$$

Here $S(p) = \hat{p}^{-1}$ is the fermion propagator, and $D_{\sigma\tau}(k) = (g_{\sigma\tau} - k_\sigma k_\tau/k^2)/k^2$ is the gluon propagator in the given approximation, in the Landau gauge. A maximum of four γ matrices can appear in the expression for the integral $\Gamma_{\mu\nu\alpha}(p,q)$. The result is

$$(16\pi^2)^2\Gamma^{\mu\nu\alpha}(p,q) = 144(q^\mu\gamma^\nu\gamma^\alpha\hat{p}\hat{q} + (\mu \to \nu)) + \text{other structures}$$

(it was for the calculation of this result that we used a dimensional regularization, although the value found does not depend on the regularization at all).

Collecting all the results, we write the following expression for the amplitude F_1:

$$\lim_{q^2\to\infty} q^2 F_1 = \pi^{-2}(1 + C_F\frac{\tilde{\alpha}_s}{\pi}), \qquad (6)$$

where $\Sigma_{\alpha=1}^{8} t^\alpha t^\alpha = C_F 1$.

Going back to the normalization of the axial current, we consider the amplitude $\langle q|J_\alpha^5|q\rangle$, where $|q\rangle$ is the state of the quark in Fok space.

Convolving this expression with γ_α, we find

$$\gamma^\alpha(< q|J_\alpha^5|q >^{\text{one loop}} - < q|J_\alpha^5|q >^{\text{tree}}) = 0.$$

We hope to discuss phenomenological consequences of result (6) in a separate publication.

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