Size of the bootstrap current in a tokamak

V. D. Pustovitov
I. V. Kurchatov Institute of Atomic Energy, 123182, Moscow

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The bootstrap current is calculated for a tokamak of circular cross section with $T_e = T_i$ in the Hirshman model, which incorporates effects of a finite aspect ratio. Upper and lower estimates of the neoclassical bootstrap current are derived.

Sustaining the current in a tokamak is a crucial problem, since success here will allow steady-state operation of tokamak systems. In this regard much hope is pinned on the bootstrap current, which neoclassical theory predicts can constitute a large fraction of the total current when the plasma pressure is high. There are also other reasons for the recent increase in interest in this effect. The conditions required for the generation of a bootstrap current are already being met experimentally.[1–4] In the JT-60 tokamak, for example, discharges in which up to 80% of the current is estimated to correspond to a bootstrap current have been produced.[4]

An important characteristic of the bootstrap current is its total magnitude J_b. In the banana regime we have the following expression[2] for J_b:

$$J_b = 0.67\beta_p\epsilon^{0.5}J_p, \tag{1}$$

where ϵ is the inverse aspect ratio, $\beta_p = 8\pi\bar{p}/B_p^2$, p is the plasma pressure, B_p is the poloidal field at the plasma edge, and J_p is the total current. This expression was derived for certain specified profiles of the temperature and the density under the condition that the current density is constant over radius. These restrictions, primarily the last, lead one to question whether expression (1) would be applicable in the general case. Yet another fundamental shortcoming (at least from the formal standpoint) of expression (1) is that is was derived from equations which are, strictly speaking, valid only at very small values of ϵ (Ref. 4). Below we derive an expression for J_b which incorporates a variation in the current density, which corresponds to more general density and temperature distributions, and which is valid at realistic values of ϵ.

To solve this problem we start from the equation

$$< \vec{j}_b\vec{B} >= 2\pi\frac{L_{31}}{n_eT_e}\left[p' + \alpha_i n_i T_i' + \frac{L_{32}}{L_{31}}n_eT_e'\right], \tag{2}$$

where the coefficients L_{3k} and α_i were calculated in Ref. 5 for a tokamak of arbitrary aspect ratio. Here $p = n_eT_e + n_iT_i$ is the plasma pressure; n_e, n_i, T_e, and T_i are the electron and ion densities and temperatures, respectively; and the prime means the derivative with respect to the total poloidal flux $\psi = \int 2\pi B_z r dr$.

Following Ref. 2, we consider tokamaks with a plasma of circular cross section. We specify the density and temperature profiles as follows ($n_e = Zn_i$, $T_e = T_i$):

$$n = n_0(1-\rho^2/a^2)^{\alpha_n}, \quad T = T_0(1-\rho^2/a^2)^{\alpha_T}. \tag{3}$$

With $\alpha_n = 0.5$ and $\alpha_T = 1.5$, these profiles become the same as the corresponding profiles in Ref. 2. Transforming the left and right sides of Eq. (2) for this case, we find the following equation for J_b:

$$\frac{J_b'}{J_p} = -\frac{4\pi p'}{B_p^2}\frac{q}{q_a}\Pi, \tag{4}$$

where q is the safety factor, q_a is the value of q at the plasma edge,

$$\Pi = L_{31}^e\left[1 + \xi\frac{\alpha_i + ZL_{32}/L_{31}}{Z+1}\right], \tag{5}$$

$L_{31}^e = L_{31}/j_0$ (in the notation of Ref 5), $\xi = \alpha_T/(\alpha_n + \alpha_T)$, and Z is the ion charge.

We can draw certain conclusions about the nature of the solution of Eq. (4) from the structure of this equation. Clearly, J_b must be proportional to J_p and β_p. This circumstance is reflected correctly in (1). The proportionality factor must be very sensitive to the profile of q, as we see from (4): With decreasing value of the ratio q_0/q_a, where q_0 is the value of q at the axis, the ratio J_b/J_p decreases. This dependence is not found in (1), which was derived for the case $q_0/q_a = 1$. The values of L_{32}/L_{31} and α_i in (5) are negative, and $0 \leqslant \xi \leqslant 1$, so Π can be written

$$\Pi = \Pi_{\max} - \xi(\Pi_{\max} - \Pi_{\min}) . \tag{6}$$

We can thus write a solution of (4) in a similar form:

$$J_b = J_b^{\max} - \xi(J_b^{\max} - J_b^{\min}) . \tag{7}$$

This expression clearly demonstrates yet another property of the bootstrap current: At a fixed pressure profile, the current J_b is higher in a plasma with a flatter temperature profile (at relatively small values of ξ).

The quantities L_{3k} and α_i in (5) are known functions of Z and of the ratio of the relative numbers of trapped and passing particles, f_t/f_c, at the given magnetic surface.[5] In the standard treatment, based on the small value of f_t/f_c, in tokamak theory, we would have $\Pi \sim \sqrt{\epsilon\rho/\alpha}$. A factor of $\epsilon^{0.5}$ would therefore appear in (1). However, even at $\epsilon = 0.1$—i.e., in practice, near the tokamak axis (the usual values are $\epsilon = 0.25$–0.4)—the ratio f_t/f_c is on the order of unity according to Ref. 5, and at $\epsilon = 0.3$ we would have $f_t/f_c > 2.5$. The approximation of a large aspect ratio $(f_t/f_c \cong 1.46\sqrt{\epsilon\rho/\alpha \ll 1})$ is of course unsuitable here. The incorporation of effects of a finite aspect ratio in Hirshman's theory turns the dimensionless factor Π in (4) into a complex function of the radius. This circumstance should ultimately be manifested in an ϵ dependence of J_b different from that in (1).

In general, Eq. (4) cannot be integrated analytically for the ϵ values typical of tokamaks. The results calculated for a plasma with $\alpha_n + \alpha_T = 1.5$ and a parabolic q profile over the interval $0.1 \leqslant \epsilon \leqslant 0.4$ can be approximated well by the simple formulas

$$\frac{J_b^{max}}{J_p} = 1.28\beta_p\epsilon^{0.34}\left[1 - 0.58\left(1 - \frac{q_0}{q_a}\right)\right], \tag{8}$$

$$\frac{J_b^{\min}}{J_p} = 1.08\beta_p\epsilon^{0.85}\left[1 - 0.51\left(1 - \frac{q_0}{q_a}\right)\right], \tag{9}$$

which can be compared with (1) quite easily. Expressions (8) and (9) were derived for a plasma with $Z = 1$. Calculating $J_b^{\max}$ and $J_b^{\min}$ for the given pressure profile completely solves the problem of finding J_b for a plasma with an arbitrary ratio α_n/α_T [see (7)].

The essential differences between (8) and (9), on the one hand, and (1), on the other, are obvious. It turns out that discarding the condition $\sqrt{\epsilon} \ll 1$ and incorporating the variation in the current density (the radial variation of q) are of fundamental importance in calculations of the bootstrap current.

The quantities $J_b^{\max}$ and $J_b^{\min}$ introduced above impose bounds on the interval of possible values of the "neoclassical" bootstrap current. Calculations show that this interval is not very wide. In the case at hand, with $\epsilon = 0.3$–0.4, the value of $J_b^{\max}$ is only twice $J_b^{\min}$ [see (8) and (9)]. The difference between $J_b^{\max}$ and $J_b^{\min}$, which depends on the pressure profile, reaches a maximum for a plasma with $Z=1$. With increasing Z, it decreases, and it vanishes at $Z \gg 1$. These consequences of the neoclassical theory are also valid for a "noncircular" plasma. Certain theoretical models predict values of J_b larger[6] or smaller[7] than the neoclassical value. If those predictions are correct, J_b may undergo "excursions" outside the interval $[J_b^{\min}, J_b^{\max}]$. The observation of such excursions experimentally would be decisive evidence in favor of one theory or another.

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