Excitation of voltage oscillations by a steady-state current in thin conductors with open Fermi surfaces

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Low-frequency oscillations of the electric field are excited by a steady-state current in a thin conductor with an open Fermi surface. The effect stems from an influence of the magnetic field of the current on the dynamics of electrons moving along open orbits.

When a thin metal plate, whose thickness d is much smaller than the electron mean free path l, is in a static, uniform external magnetic field $\mathbf{H}_0$, its electrical conductivity is sensitive to the topological structure of the electron energy spectrum and also to the surface state of the conductor. Electrons in a metal whose Fermi surface is a corrugated cylinder with a p_y axis move in a magnetic field $\mathbf{H} = (0,0,H_z)$ along an open orbit along the x axis in coordinate space. The motion of such electrons in a plate with specular-reflection faces at $x_s = \pm d/2$ is strictly periodic. The displacement (δ) of an electron along the current direction (the y axis), as the electron moves from one surface of the plate to the other, depends on the magnetic field. In particular, at a certain value $H_0 = H_N$ the electron traverses a distance d in an integer number of periods of motion along the open orbit, $r_0(H_0)$. The electrons which belong to open cross sections of the Fermi surface contribute little to the electrical conductivity, because their drift along the current direction results exclusively from interior collisions.[1]

At high current densities $\mathbf{j}$, the effect of the magnetic field of the current, $\mathbf{H}_j$, on the dynamics of the electrons must be taken into account. In the nonlinear regime, the electrical conductivity of a plate may either increase or decrease with increasing voltage, depending on the strength of the resultant magnetic field $\mathbf{H} = \mathbf{H}_0 + \mathbf{H}_j$. There can be a situation in which an increase in the voltage leads to a change in H_j such that δ decreases, so there is also a decrease in the contribution of electrons on open cross sections of the Fermi surface to the current. On the other hand, the current distribution is unstable at values of H at which δ is close to zero, since slight changes in j and H lead to an increase in the drift of electrons and thus to an increase in the current (j_{0p}) carried by electrons moving along open orbits. Similar effects should occur in cases in which the thickness of the shell of open orbits is not too small, and the current j_{0p} is comparable to the current of the electrons in closed orbits. Below we show that a quasiperiodic dependence of the electrical conductivity on the magnetic field H leads to an excitation of low-frequency oscillations of the electric field by a steady-state current.

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To find the current density and the electric potential, we work from a system of equations consisting of the Boltzmann kinetic equation and Maxwell's equations. We assume that the scattering of the electrons by the boundary of the sample is specular. We assume a dispersion relation

$$\epsilon(\vec{p}) = \frac{p_x^2 + p_z^2}{2m} + \epsilon_0 \cos\frac{p_y}{p_0}, \tag{1}$$

where ϵ_0 satisfies the condition $(d/l)^2\epsilon_F \ll \epsilon_0 \ll \epsilon_F$, where ϵ_F is the Fermi energy. A Fermi surface in the form of a slightly corrugated cylinder is apparently a good description of organic metals.[2] If we neglect the heating of the charge carriers, we should linearize the kinetic equation in terms of the electric field **E**. The magnetic field is found from the equation

$$-\frac{dH_z(x)}{dx} = \frac{4\pi}{c} j_y(x). \tag{2}$$

Under the assumptions made above, we can write the current density and the electric potential as power series in the small parameters ϵ_0/ϵ_F and $(d/l)(\epsilon_F/\epsilon_0)^{1/2}$. The leading asymptotic term of the current density is

$$j_y(x) = I_o \int_{-1/2}^{1/2} d\xi' \cos\frac{e}{cp_0}(A_y(\xi) - A_y(\xi')), \tag{3}$$

where $A_y(x) = \int_0^x dx' Hz(x')$, $I_0 = (\epsilon_0/\epsilon_F)^2\sigma_0 E_y$, $\sigma_0 = e^2 n_0 \tau/m$, $n_0 = (m\epsilon_F)^2/4\pi h^3 p_0$, $\xi = x/d$, $\mathbf{A}(x)$ is the vector potential, e is the charge of an electron, c is the velocity of light, τ is the mean free time of an electron, and $\hbar$ is Planck's constant. The constants of integration for (2) and (3) are determined by specifying the magnetic field H_0 and by also specifying either the external voltage or the total current in the conductor, J. A solution of Eqs. (2) and (3) is

$$A_y(\xi) = 2A_0\left\{am\left(\eta(H_0)\Omega\xi + K\left(\frac{\theta}{2},\mu\right),\mu\right) - \frac{\theta}{2}\right\}. \tag{4}$$

Here

$$\eta(H_0) = \begin{cases} 1 H_0 > 0 \\ -1 H_0 \le 0, \end{cases} \quad K\left(\tfrac{\theta}{2},\mu\right) = \int_0^{\theta/2} dt(1-\mu^2\sin^2 t)^{-1/2},$$

$am(\zeta,\mu)$ is the amplitude of the elliptic integral, $\zeta = K(am\zeta,\mu)$, $\Lambda_0 = h_0 d$, $h_0 = p_0 c/ed$ is the magnetic field in which the spatial period of the motion along an open orbit is $2\pi d$,

$$\mu^2 = \frac{\beta}{\Omega^2},\ \Omega^2 = \frac{1}{4}\frac{H^2(0)}{h_0^2} + \beta\sin^2\frac{\theta}{2},\ \beta = \lambda^{1/2}\left(\frac{I}{j_0}\right)^{1/2},\ \lambda = \frac{I_0}{j_0}, \tag{5}$$

$j_0 = ch_0/4\pi d$, and the parameters I and $\sin(\theta/2)$ are given by

$$I=\int_{-1/2}^{1/2} d\xi' j_y(\xi'),$$

$$\sin^2\frac{\theta}{2}=\frac{1}{2}\left(1+\frac{1}{2\beta h_0^2}\int_{-1/2}^{1/2} d\xi'(H^2(\xi')-H^2(0))\right). \qquad (6)$$

Equations (5) and (6) are transcendental equations which implicitly specify the relationship between E_y and $J=DdI$, where D is the width of the plate. This width satisfies $D\gg d$. The distributions of the current density and the magnetic field are expressed in terms of elliptic functions. They depend strongly on the value of μ, which is in turn determined by the average values of $j_y(\xi)$ and $H^2(\xi)$ over the thickness of the sample.

Let us analyze the stability of Eqs. (2) and (3) with respect to small space-time perturbations of the type

$$A_y(x,t)=A_y(x)+\tilde{a}(x)e^{-i\omega t+ikx},\quad E_y(x,t)=E_y+i\frac{\omega}{c}\tilde{a}(x)e^{-i\omega t+ikx}. \qquad (7)$$

Substituting (7) into (2), (3), and linearizing in terms of the small perturbations, we find that the function $a(x)=\tilde{a}(x)\exp(ikx)$ satisfies the Hill equation

$$\frac{d^2a(\xi)}{d\xi^2}=-\left[\frac{i\omega p_o}{e}\frac{\partial\beta}{\partial E_y}\sin A_1(\xi)+\beta\cos A_1(\xi)\right]a(\xi),$$

$$A_1(\xi)=2am(\eta(H_0)\Omega\xi+K(\frac{\theta}{2},\mu),\mu). \qquad (8)$$

We find the relationship between the frequency ω and the wave vector k by setting the Hill determinant equal to zero. If μ is not too close to one, we can find explicit expressions for ω as a function of k in the form of power series in the small parameter $\gamma=\exp(-\pi k)$:

$$\omega^2=\omega_0^2\frac{\left[\sin^2\left(\frac{2K(1/\mu)kd}{\sqrt{\beta}}\right)-\sin^2\pi\sqrt{F_0}\right]}{4\pi\sin 2\pi\sqrt{F_0}}\cosh^2\left(\frac{\pi}{2}\kappa\left(\frac{1}{\mu}\right)\right)\sqrt{F_0}(4F_0-1)$$
$$+O\left(\omega_0^2\exp\left(-\pi\kappa\left(\frac{1}{\mu}\right)\right)\right), \qquad (9)$$

where

$$\omega_0=\frac{e}{p_0}\beta\frac{\partial E_y}{\partial\beta},\quad F_0=\frac{4}{\pi^2}K\left(\frac{1}{\mu}\right)\left(2\bar{E}\left(\frac{1}{\mu}\right)-K\left(\frac{1}{\mu}\right)\right),\quad K(q)\equiv K\left(\frac{\pi}{2},q\right),$$

$$\bar{E}(q) = \int_0^{\pi/2} dt\sqrt{1 - q^2 \sin^2 t}, \quad \kappa(q) = K(\sqrt{1-q^2})/K(q),$$

for $\mu > 1$ and

$$\omega^2 = -\omega_0^2 \left\{ \frac{\left[\sin^2\left(\frac{K(\mu)kd}{\Omega}\right) + \sinh^2 \pi\sqrt{\Psi_0}\right]}{\pi \mathrm{sh} 2\pi\sqrt{\Psi_0}} \sqrt{\Psi_0}(4\Psi_0 + 1) \right.$$

$$\left. - \frac{1}{\sinh^2(\pi\kappa(\mu))} \right\} \cosh^2(\pi\kappa(\mu)) + O(\omega_0^2 \exp(-2\pi\kappa(\mu))), \tag{10}$$

where $\Psi_0 = 1/\pi^2[2\kappa^2(\mu) - \mu^2 K(\mu) - 2\bar{E}(\mu)K(\mu)]$, for $\mu < 1$. If $\kappa \approx 1$, then $\gamma \approx 4.3 \times 10^{-2}$.

There is a broad range of values of J, d, and H_0 in which the oscillation frequency in (9) is real and lies in the range 10^3–10^5 Hz. At $\mu < 1$, the frequency in (10) is imaginary, and no oscillatory processes occur. If there is no emission of electromagnetic waves into the space outside the plate, then the thickness of the sample, d, must be equal to an integer number of half-wavelengths: $k = \pi n/d$, $n = \pm 1, \pm 2 \ldots$. The oscillations arise at current densities at which the quantity $(\lambda I/j_0)^{1/4}$ reaches a value on the order of one. A consequence of the strong nonlocal effects, which are manifested by a dependence of the modulus (μ) of the elliptic functions $j_y(\xi)$ and $H(\xi)$ on their integrals over ξ [see (5) and (6)], is a pronounced sensitivity of the current distribution to changes in the magnetic field strength. Equations (5) and (6) show that the H_j dependence of μ must also be taken into account in the case $H_j \ll H_0$. At a given current J, we can work from the condition $m > 1$ to estimate the maximum value of the external magnetic field at which oscillatory processes can still occur: $H_0 < \sqrt{2}(\lambda I/j_0)^{1/4} h_0$.

The nonlinear effect discussed above weakens with decreasing value of the specular parameter $\bar{p}$, but it remains substantial at $\bar{p} \gg d/l$.

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