Existence of an effective magnetic field in disordered many-valley semiconductors

D. G. Polyakov
A. F. Ioffe Physico-Technical Institute, Academy of Sciences of the USSR, 194021, Leningrad

(Submitted 13 August 1991)
Pis'ma Zh. Eksp. Fiz. **54**, No. 7, 401–404 (10 October 1991)

An internal effective magnetic field is shown to affect an electron when it moves in the valley of a silicon-like crystal in the presence of a random static potential. The phase relaxation due to the scattering by a randomly inhomogeneous effective magnetic field is considered.

Let us assume that the electron spectrum of a crystal has several degenerate extrema located inside the Brillouin zone (not on its boundary). Let us also assume that the intervalley scattering by a random static potential is weak compared to the intravalley scattering. In this letter, we should like to call attention to the existence of an effective magnetic field with which an electron interacts when it diffuses inside one of the valleys. It is convenient to elucidate this process by assuming the inhomogeneities to be smooth on the scale of a wavelength. The point is that the position in $\vec{k}$-space of the band extremum is then not the same in different regions of the crystal. If the wave vector $\hat{k}_0$ which defines the bottom of the valley depends on $\vec{r}$, the function $(-\hbar c/e)\vec{k}_0(\hat{r})$ plays the role of a vector potential, while $(-\hbar c/e)\mathrm{curl}\vec{k}_0(\vec{r})$ of the magnetic field. The existence of an effective magnetic field due to the spatial variation of $\vec{k}_0$ has been suggested by Kroemer[1] many years ago as applied to a similar object: a solid solution with smoothly varying chemical composition.

Thus, corresponding to each valley is its own magnetic field. According to the time symmetry, the fields of the valleys which differ in $\vec{k}_0$ sign are opposite in direction. We consider the action of an effective magnetic field which is caused by a random electric field or a random lattice deformation and which fluctuates in space on a scale much lower than the mean free path. Typically, the time of the intervalley transitions in degenerate semiconductors with a multiply connected Fermi surface at low temperatures is much greater than the momentum relaxation time, which is assumed in what follows.

 © 1991 American Institute of Physics

For definiteness, we mean a silicon crystal whose conduction band consists of six valleys located on [100] axes; value of k_0 is $\simeq 0.85 \cdot 2\pi/a$, and a is the lattice constant. The existence of an effective magnetic field implies that the term H_M in the band Hamiltonian is linear in the wave vector $\vec{k}$, reckoned from the valley bottom:

$$H_M = \frac{\hbar}{2}(\vec{k}\vec{v}(\vec{r}) + \vec{v}(\vec{r})\vec{k}).$$

The relationship between the velocity $\vec{v}$ and the electric field $\vec{E}$, which is not too strongly inhomogeneous, can be established by making use of the $\vec{k}\vec{p}$ theory to fourth order. It is given by

$$\vec{v}(\vec{r}) = \alpha\frac{\vec{k}_0}{k_0}\left(\frac{\partial\vec{E}}{\partial\vec{r}}\right) + \left(\frac{\vec{k}_0}{k_0}\frac{\partial}{\partial\vec{r}}\right)\hat{\beta}\vec{E} \tag{1}$$

with the three constants of an "atomic" scale: α and the independent components β_1 and β_2 of the tensor $\hat{\beta}$ of rank two.[1] The first term in (1) is available only in regions with nonzero charge density. By choosing the z axis along $\vec{k}_0$

$$v_z = \alpha\left(\frac{\partial\vec{E}}{\partial\vec{r}}\right) + \beta_1\frac{\partial E_z}{\partial z}, \quad v_x = \beta_2\frac{\partial E_x}{\partial z}, \quad v_y = \beta_2\frac{\partial E_y}{\partial z}.$$

We see that a magnetic field exists in the plane perpendicular to $\vec{k}_0$.

A nonuniform distortion of the crystal lattice causes the effective magnetic field to appear in the second order of the perturbation theory by simultaneously taking into account the $\vec{k}\vec{p}$-term and the deformation potential. Because of symmetry, the connection between $\vec{v}$ and the strain tensor ε is similar to (1) with changing $\partial E_i/\partial x_j \Leftrightarrow \varepsilon_{ij}$: $v_z = \alpha'\mathrm{Sp}\varepsilon + \beta_1'\varepsilon_{zz}$, $v_x = \beta_2'\varepsilon_{xz}$, $v_y = \beta_2'\varepsilon_{yz}$. Note that the z component of the magnetic field exists under deformation but only under that which is accompanied by rotation, provided that $\partial u_z/\partial y - \partial u_y/\partial x \neq 0$, where $\vec{u}$ is the displacement vector.[2] However nontrivial it is, the constants α', β_1', and β_2' should be determined spectroscopically by the shift of the valley minima shift with an external stress. In silicon, the proximity of $\vec{k}_0$ to the Brillouin zone boundary facilitates an indirect measurement of this shift. In the presence of nondiagonal strain component ε_{xz}, the electron spectrum contains the term $\hbar^2 k_x(k_z + k_0 - 2\pi/a)2m_{xz}$ with an effective mass $m_{xz} \propto \varepsilon_{xz}^{-1}$. If k_0 is close to $2\pi/a$, the same term gives rise to the shift of the minimum along x at $k_z = 0$, so that $\beta_2' = \hbar(k_0 - 2\pi/a)/2m_{xz}\varepsilon_{xz}$. The value of m_{xz} can be extracted from the variation of the cyclotron mass at the band edge vs uniaxial stress orientation. (Such a dependence has been obtained in Ref. 2, but the measurement accuracy was not good enough for the orientation to be of interest to us.) The difference $k_0 - 2\pi/a$ is expressed in terms of the longitudinal mass $m_\parallel$ and the velocity v_0, which defines the splitting $\hbar v_0(k_z - 2\pi/a)$, linear in k_z, of the spectrum near the Brillouin zone boundary, $k_0 = 2\pi/a - m_\parallel v_0/\hbar$. Thus, knowledge of $m_\parallel$ and v_0 as the functions of $\mathrm{Sp}\varepsilon$ and ε_{zz} will enable us to find constants α' and β_1' (see Ref. 2 for rough estimate).

In the weak localization range the interference phenomena are caused by a spatial coherence of waves that pass the loop of the diffusion trajectory in opposite direc-

tions.[3] As the temperature is lowered, even weak interactions, which do not preserve this coherence, manifest themselves. If an electron is scattered by a random magnetic field, the amplitude $f(\vec{k}_1, \vec{k}_2)$ of the $\vec{k}_1 \to \vec{k}_2$ transition is not equal to $f(-\vec{k}_2, -\vec{k}_1)$, which breaks the coherence. In the Born approximation

$$f(\vec{k}_1, \vec{k}_2) = -\frac{m}{2\pi\hbar}(\vec{k}_1 + \vec{k}_2)\vec{v}_{\vec{k}_1 - \vec{k}_2},$$

where $\vec{v}_{\vec{k}_1 - \vec{k}_2}$ is the Fourier transform of the velocity $\vec{v}(\vec{r})$, and m is the density of states of the mass. The amplitudes $f(\vec{k}_1, \vec{k}_2)$ and $f(-\vec{k}_2, -\vec{k}_1)$ will then differ in sign only. For this reason, the phase relaxation time τ_M is simply half the reciprocal probability of scattering by a random magnetic field, where τ_M is assumed to be much larger than the mean free time. Let us estimate τ_M in heavily doped semiconductors deep in the metal, where the Fourier component of the electric field correlator has the form $\langle E_i E_j \rangle_{\vec{k}} = 16\pi^2 n e^2 k_i k_j / (k^2 + r_s^{-2})^2 \varkappa^2$; here n is the charged impurity concentration, r_s is the screening radius, and $\varkappa$ is the dielectric constant. It should be noted that the dispersion is anisotropic, that the momentum relaxation time depends on the momentum orientation, and that τ_M is obtained by averaging over the wave vectors of the final and initial states. We thus find

$$\frac{1}{\tau_M} = \frac{64\pi^3}{3\hbar}\rho_F m_{\|}\epsilon_F \frac{ne^2}{\kappa^2}\left(\alpha + \beta_1 - \frac{m_\perp}{m_\|}\beta_2\right)^2,$$

where m is the density of states with the Fermi energy ε_F (allowing for one spin projection). It was taken into account that an effective mass $m_\|$ along [100] in silicon is much larger than $m_\perp$ of the transverse motion (the ratio of α, β_1, and β_2 was arbitrary). Notice an absence of Coulomb divergence when evaluating τ_M; for this reason we set $r_s = \infty$. The rate of phase relaxation increases rapidly with the doping level ($\tau_M^{-1} \propto n^2$). The intervalley transition time and τ_M may be of any ratio, depending, as they do, on the potential core strength and interband energy distances. Similarly, for the scattering in a magnetic field caused by short-range distortions (whose correlation radius is less than the Fermi wavelength),

$$\frac{1}{\tau_M} = \frac{4\pi}{3\hbar}\rho_F m_\| \epsilon_F d, \;\; d = (d_1 + 2d_2)(3\alpha'^2 + 2\alpha'\beta_1') + d_1\beta_1'^2 + 2\frac{m_\perp}{m_\|}d_3\beta_2'^2,$$

where $d_{1,2,3}$ identifies the correlators of the strain fields

$$< \epsilon_{xx}\epsilon_{xx} >_{\vec{k}} = d_1, \;\; < \epsilon_{xx}\epsilon_{yy} >_{\vec{k}} = d_2, \;\; < \epsilon_{xy}\epsilon_{xy} >_{\vec{k}} = d_3.$$

In an n-type Si-MOS structure with a (100) surface, two dimensional electron gas occupies two lowest-lying equivalent valleys oriented normal to the boundary plane. In this geometry an effective magnetic field arises from the lattice deformation only. Therefore, the phase relaxation time in high-quality Si(100) structures may be much larger than that with another surface orientation.

Thus, the interference phenomena are determined by three characteristic times: the phase relaxation time τ_E, which is associated with the inelastic processes, and the intervalley transition times τ_v and τ_M. If $\tau_v \gg \tau_M$, an experimentally measured phase-

breaking time $(\tau_E^{-1}+\tau_M^{-1})^{-1}$, which increases with decreasing temperature, should tend to saturation at the value of τ_M. Of course, an effective magnetic field, which destroys the Cooperon attributed to one valley, does not affect the Cooperon constructed with the wave functions of different valleys in a symmetrical way. Hence, at lowest temperatures, τ_E which is larger than τ_v, is the symmetrized Cooperon that governs interference phenomena. In the opposite case $\tau_v \ll \tau_M$, although an electron changes valleys that differ in $\vec{v}(\vec{r})$ sign many times in a time τ_M, the expression for τ_M remains the same (if the space fluctuations of the magnetic field are smooth enough, it will not be the case). Note that the decay of the Cooperon due to the intervalley transitions takes place only because of the nonequivalence of differently oriented valleys. If the symmetry of the valleys is the same (and τ_m^{-1} are neglected, see below), the intervalley scattering does not manifest itself at all. Correspondingly, when considering the total Cooperon time evolution in Ref. 4, only the number of nonequivalent valleys are relevant. The suggestion in Ref. 5 that the intervalley transitions in the Si(100)-MOS structure have a destructive effect is at variance with the conclusion made above. The point is that the processes of multiple intervalley scattering, which end in the same valley as the electron waves begin, were ignored in Ref. 5. For example, the expression for the Cooperon which describes the interference in two equivalent valleys (in Si-MOSFET or uniaxially strained bulk Si, for example) is proportional to the sum

$$\frac{1}{x+\tau_M^{-1}}+\frac{1}{x+\tau_M^{-1}+2\tau_v^{-1}}+\frac{1}{x}-\frac{1}{x+2\tau_v^{-1}},$$

where $x=\vec{Q}D\vec{Q}+\tau_E^{-1}$, $\vec{Q}$ is the Cooperon momentum, and D is the diffusion coefficient. If $\tau_v \gg \tau_M$ [and $x \gg (\tau_M\tau_v)^{-1/2}$], this sum reduces to $2/(x+\tau_M^{-1})$, and the temperature dependence is cut off at $\tau_E \sim \tau_M$. If $\tau_v \ll \tau_M$, then this expression becomes $(x+\tau_M^{-1})^{-1}+x^{-1}$ and half of the Cooperon retains the pole form.

I am grateful to A. G. Aronov, G. E. Pikus, and B. Z. Spivak for interesting discussions.

[1] Since the crystal has no inversion symmetry, an interaction similar to the spin-orbit interaction, where $\vec{k}_0$ is substituted for the spin vector, should exist.

[2] It is interesting that the Burgers dislocation oriented along $\vec{k}_0$ induces the magnetic field in the same way as the solenoid does. Being zero everywhere except on the dislocation axis, such a field is responsible for the Aharonov–Bohm effect.

[1] H. Kroemer, RCA Rev. **18**, 332 (1957).
[2] J. C. Hensel, H. Hasegawa, and M. Nakayama, Phys. Rev. **138**, A225 (1965).
[3] B. L. Altshuler, A. G. Aronov, D. E. Khmel'nitskii, and A. I. Larkin, in: *Quantum Theory of Solids*, ed. I. M. Lifshitz (Mir, Moscow, 1982).
[4] B. L. Altshuler and A. G. Aronov, in: *Electron-Electron Interactions in Disordered Systems*, ed. A. L. Efros, and M. Pollak (North-Holland, Amsterdam, 1985).
[5] H. Fukuyama, *ibid.*

Submitted in English by Author