Impurity-assisted tunneling in a quantum ballistic microconstriction

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We calculate the conductance of a pinch-off saddle-point potential ballistic microconstriction with a short-range impurity in the conducting channel. Resonant tunneling via impurity bound states causes narrow peaks in conductance vs Fermi energy.

Recent experiments demonstrate that even a single impurity can affect the conductance of a quantum ballistic microconstriction.[1] The effect of the impurity is especially strong near the thresholds which correspond to steps between adjacent quantization plateaus. This result is confirmed by calculations.[2,3] It follows from these calculations that in the case of a short-range attractive impurity a bound state appears below each threshold. Resonant scattering by the bound state causes a narrow dip in conductance G vs Fermi energy E. In Refs. 2 and 3 the microconstriction was modeled as an infinite waveguide with a constant cross section. We adopt the saddle-point potential,[4,5] which is a more realistic model.[6,7] In this model bound states exist also for repulsive impurities, and resonant scattering displays not only dips in G vs E, but also peaks. The peaks are due to electron resonant tunneling via bound states.

The saddle-point potential is assumed to be

$$V(x,y)=\frac{\hbar^2}{2md^2}\left[-\frac{x^2}{L^2}+\frac{y^2}{d^2}\right]. \tag{1}$$

Here d is the width of the channel, L is its length, d is of the order of the Fermi wavelength λ_F, and $L \gg d$. The waveguide modes for the potential (1) are

$$\Psi^{\pm}_{E,n}(x,y)=\Phi_n(y)E(-\varepsilon_n,\pm\xi). \tag{2}$$

Here E is the electron energy, Φ_n $(n=0,1,...)$ are the harmonic oscillator wave functions which correspond to the energies $E_n=\hbar\Omega(n+\frac{1}{2})$ with $\hbar\Omega=\hbar^2/md^2$, $E(-\varepsilon,\xi)$ is the Weber function[8] with $\varepsilon_n=(E-E_n)/\hbar\omega$ and $\hbar\omega=\hbar^2/mdL$, and $\xi=x(2/Ld)^{1/2}$. The functions $\Psi^{\pm}_{E,n}$ correspond to the modes n coming from $x=\mp\infty$. The threshold energy for mode n is E_n.

When there is no impurity, the conductance of microjunction (in units of $2e^2/h$) is[5]

$$G_0=\sum_{n=0}^{\infty}t^2(\varepsilon_n). \tag{3}$$

with the transmission coefficient

$$t^2(\varepsilon) = 1 - r^2(\varepsilon) = (1 + \exp(-2\pi\varepsilon))^{-1}. \tag{4}$$

The impurity changes the conductance due to mixing and additional reflection of modes. Following Refs. 2 and 3, we assume that the impurity is a short-range impurity. The scattered field for such an impurity is

$$\psi'(r) = -2\pi\psi^\circ(r_0)\frac{G_E(r,r_0)}{D_E(r_0)}. \tag{5}$$

Here r_0 is the impurity position, $\psi^0(r)$ is the field in absence of the impurity, and $G_E(r,r')$ is the Green's function of the Schrödinger equation with the confining potential $V(x,y)$. The denominator D_E is expressed in terms of the nearly asymptotic behavior of the Green's function

$$G_E(r,r')|_{r,r'\to r_0} = \frac{1}{2\pi}\left[\ln\frac{d}{|r-r'|} + K_E(r_0)\right]; \tag{6}$$

specifically,

$$D_E(r_0) = \Lambda + K_E(r_0). \tag{7}$$

Here $\Lambda = \ln(d/a)$, where a is the 2D scattering length of the impurity potential.

Calculating the scattered field for $\psi^0 = \Psi^+_{En}$, we find the transmission coefficients $T_{n\to n'}$ and the conductance

$$G = \sum_{nn'}|T_{n\to n'}|^2. \tag{8}$$

In what follows we consider the case of a pinch-off microconstriction, $E < E_0$, and assume the impurity to be in the narrow part of the constriction, $x_0 \ll L$. In this simple case

$$K_E = p + \beta H(\varepsilon,\xi_0), \tag{9}$$

where

$$H(\varepsilon,\xi) = \frac{1}{\sqrt{2}}t(\varepsilon)E(-\varepsilon,\xi)E(-\varepsilon,-\xi) \equiv P(\varepsilon,\xi) + iQ(\varepsilon,\xi) \tag{10}$$

with $\varepsilon \equiv \varepsilon_0$ and $\beta = \pi(L/d)^{1/2}\Phi_0^2(y_0)$. Here βH is the singular contribution due to the threshold mode $n = 0$. The contribution due to below-barrier modes $n > 0$ is given by the constant p, which is real and of the order of unity. The conductance of a pinch-off constriction reduces to

$$G = G_0\frac{|\tilde{\Lambda}|^2}{|D_E|^2}, \tag{11}$$

where $G_0 = t^2(\varepsilon)$ is the conductance in the absence of the impurity, $\tilde{\Lambda} = \Lambda + p$, and $D_E = \tilde{\Lambda} + \beta H$.

The sharp features on the G-vs-E curve occur because of the Breit–Wigner resonances,[9] which correspond to resonance scattering by the impurity bound states. The bound states are defined as scattering-amplitude poles $\bar{E} - i\Gamma$ which are located near the real E axis. These poles are roots of the equation $D_E = 0$. The width of the bound state Γ is small in the energy domain, where $Q \ll P$. That is why $\bar{E}$ can be sought from the equation

$$\tilde{\Lambda} + \beta P = 0, \tag{12}$$

and also

$$\Gamma = \left(\frac{Q}{dP/dE}\right)_{E=\bar{E}}. \tag{13}$$

If one develops D_E near the pole, (11) will reduce to

$$G(E) = G_0(E)\left(\frac{P}{Q}\right)^2_{E=\bar{E}} \cdot \frac{\Gamma^2}{(E-\bar{E})^2 + \Gamma^2}. \tag{14}$$

It follows from this expression that each impurity bound state manifests itself as a peak on the G-vs-E curve, where the peak value of the conductance with an impurity is larger by a factor of $(P/Q)^2$ than the conductance without an impurity.

The investigation of the bound states and the conductance peaks can be greatly simplified, since $Q \ll P$ only if $\varepsilon < 0$ and $|\varepsilon| \gg 1$. In this energy domain the conductance (14) can be represented in the form

$$G(E) = \frac{4\Gamma'\Gamma''}{(E-\bar{E})^2 + \Gamma^2}. \tag{15}$$

Here $\Gamma = \Gamma' + \Gamma''$, where Γ' and Γ'' are partial widths due to the escape of the electron from the bound state to $x = +\infty$ and $x = -\infty$, respectively. Now it is obvious that the peaks on the G-vs-E curve are due to resonant tunneling via the impurity bound state. To find the bound states explicitly, we express the Weber functions in terms of Airy functions.[8] The situation is relatively simple in the case where the impurity is in the central cross section, $\xi_0 = 0$. Only one bound state exists, and only in the case of a strong enough attractive impurity, i.e., when $\tilde{\Lambda} < 0$ and $|\tilde{\Lambda}| \ll \beta$. The binding energy and the width are

$$\Delta = E_0 - \bar{E} = \hbar\omega\frac{\beta^2}{2|\tilde{\Lambda}|^2}, \qquad \Gamma' = \Gamma'' = \Delta \exp(-\pi\Delta/\hbar\omega). \tag{16}$$

Consider now the more complicated situation, when the impurity is displaced from the central cross section, $\xi_0 \gg 1$. In this case bound state exists if $|\tilde{\Lambda}| \ll \beta\xi_0^{-1/3}$, and two types of such states appear. The first type of bound state is of the same nature as that for the central impurity and exist only for attractive impurities. The corre-

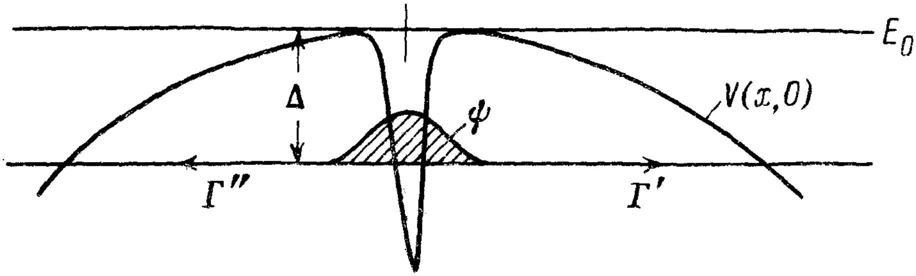

FIG. 1.

sponding energy level is below the level $E_0 + V(x_0,0)$. The binding energy, if reckoned from this level, coincides with Δ given by (16) (Fig. 1). The second type of bound state exists for both signs of the impurity potential. The energy levels are above the level $E_0 + V(x_0,0)$, where the wave functions are being "mirror-confined" waves reflected from the saddle-point potential on the left and from the impurity potential on the right (Fig. 2). The number of mirror-confined states is of the order of ξ_0^2.

To avoid dealing with a complicated formula, we assume in what follows that $|\widetilde{\Lambda}| \gg \beta\xi_0^{-1}$. Then the partial widths of the first type bound state are

$$\Gamma' = \Delta \exp(-\sigma^3/3),$$

$$\Gamma''/\Gamma' = G_0(\bar{E})(\Delta/\Gamma')^2 \ll 1, \tag{17}$$

where $\sigma = \sqrt{2}\beta/|\widetilde{\Lambda}|\xi_0^{1/3}$. Both Γ′ and Γ″ are due to the tunneling through wide barriers and hence are exponentially small.

The position of the "mirror-confined" energy levels is given by

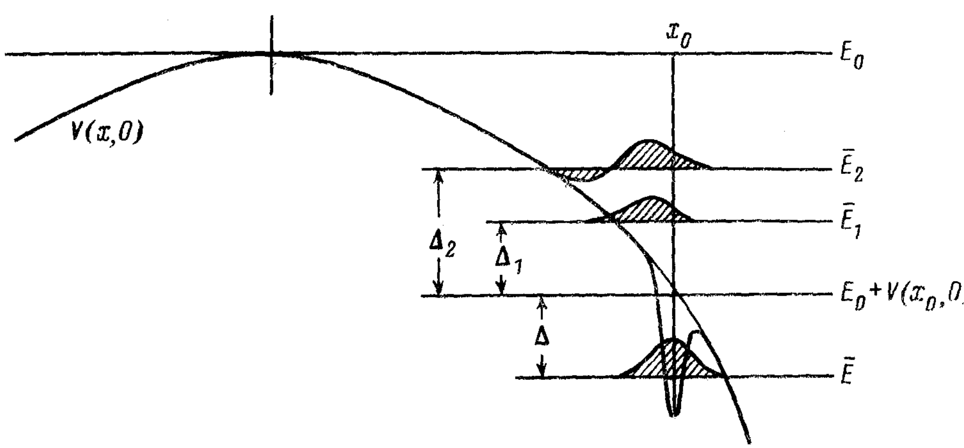

FIG. 2.

$$\Delta_s = \hbar\omega(\xi_0/2)^{2/3} t_s, \tag{18}$$

where t_s is a zero of the Airy function: $Ai(-t_s) = 0$. The partial widths are

$$\Gamma'_s = c_1 \Delta_s/\sigma^2,$$

$$\Gamma''_s/\Gamma'_s = c_2 G_0(\bar{E})(\Delta_s/\Gamma'_s) \ll 1, \tag{19}$$

where

$$c_1^{-1} = 2^{2/3}\pi t_s Bi^2(-t_s), \qquad c_1 c_2 = t_s^{-2}. \tag{20}$$

The right barrier created by the impurity is narrow and hence Γ'_s is not exponentially small.

Introducing (17) or (19) into (15), we can calculate the enhancement of the conductance due to the impurity-assisted resonant tunneling.

[1]G. L. Timp and R. E. Howard, to appear in Proc. IEEE, Jan. 1991.
[2]C. S. Chu and R. S. Sorbello, Phys. Rev. B**40**, 5941 (1989).
[3]P. Bagwell, Phys. Rev. B**41**, 10354 (1990).
[4]A. H. Fertig and B. I. Halperin, Phys. Rev. B**36**, 7969 (1987).
[5]M. Buttiker, Phys. Rev. B**41**, 7906 (1990).
[6]S. E. Laux, D. J. Frank, and F. Stern, Surf. Sci. **196**, 101 (1988).
[7]K. F. Berggren, G. Roos, and H. van Houten, Phys. Rev. B**37**, 10118 (1988).
[8]Handbook of Mathematical Function, ed. M. Abramowitz and I. A. Stegun, NBS (1964).
[9]M. Buttiker, IBM J. Res. Develop. **32**, 63 (1988).

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