New superconformal string

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An operator formalism is used to derive N-particle tree amplitudes for a new conformal string model. In contrast with classical string theories based on the dynamics of $2D$ boson and fermion fields, this new superconformal dynamics is determined exclusively by $2D$ fermion fields. For these fermion modes, the Pauli principle makes the spectrum of physical states much narrower than that in classical string theories. The Pauli principle also makes it possible to compare this state spectrum with the hadron spectrum in π-meson interactions.

We would like to describe a new class of string models, in which it is possible to achieve a correspondence between the string spectrum and the hadron spectrum while retaining the superconformal symmetry of the string amplitudes. Two-dimensional fermion fields and two operators, X_0 and its conjugate P (a momentum), which correspond to the motion of the string as a whole, are involved in the derivation. As in the standard approach in the operator formalism,[1] we represent the tree string amplitudes as the vacuum expectation value of the product of operator vertices $V(\tau_i,k_i)$ for the emission of particle i, with a momentum k_i (open strings):

$$A_N = \int \prod_{i=2}^{N-1} d\tau_i < 0|V(\tau_1)V(\tau_2)...V(\tau_N)|0$$

with

$$V(\tau) = I(\tau)\exp(ikX(\tau)) = \exp iL_0\tau V(0)\exp(-iL_0\tau) \qquad (1)$$

(Ref. 2). Ordinary Lorentz invariance and the zero conformal dimensionality for the operator $X_\mu(\tau)$ in the absence of primary boson fields require the derivation of a composite 4-vector $X_\mu(\tau)$ from fermion fields. For this purpose we introduce anticommuting fields $\psi_\alpha(\tau)$ with a Dirac spinor index $\alpha(\alpha = 1,2,3,4)$. Since we have $\psi_\alpha = \Sigma_n \psi_{n\alpha} e^{-in\tau} = \psi_\alpha^+$, and since the vector current $\partial_\tau X_\mu$ is odd in charge, we need two Majorana fields ψ for this current. Assuming that these components are isotopic, we obtain an eight-component spinor and an isospinor $\psi_{\alpha\gamma}$ $(\gamma = 1,2)$, with the usual anticommutation relations for the components of the fields of conformal dimensionality 1/2:

$$\{\tilde{\psi}_{n\alpha\gamma}, \psi_{m\beta\delta}\} = \delta_{\alpha\beta}\delta_{\gamma\delta}\delta_{n_1-m} \qquad \tilde{\psi} \equiv \psi\gamma_0\tau_2. \qquad (2)$$

From these eight components of the field $\psi_{\alpha\gamma}(\tau)$, we can form 28 components of corresponding currents, whose components obey a Kac–Moody algebra:

$$J^V_\mu(\tau) = \tilde{\psi}\gamma_\mu\psi; \qquad J^A_{\mu i}(\tau) = \tilde{\psi}\gamma_5\gamma_\mu\tau_i\psi; \qquad J^s_i = \tilde{\psi}\tau_i\psi;$$

$$J^P_i = \tilde{\psi}\gamma_5\tau_i\psi; \quad J^T_{\mu\nu} = \frac{1}{2}\tilde{\psi}[\gamma_\mu, \gamma_\nu]\psi \ . \tag{3}$$

Here γ_μ are the Dirac matrices in the Majorana picture, and τ_i are isotopic Pauli matrices. In order to obtain Lorentz-invariant and isotopically scalar trilinear combinations of components of the fermion fields for the operators of a superconformal Virasoro algebra,[3] we introduce some new anticommuting fields $\varphi_\mu(\tau)$, $\kappa_{i\mu}(\tau)$, $\eta_i(\tau)$, $\vartheta_i(\tau)$, and $\zeta_{\mu\nu}(\tau)$, with the quantum numbers of the currents written above [see (3)]. These new fields obey the standard anticommutation relations for free fields [see (2)]: $\{\varphi_{n\mu}, \varphi_{m\nu}\} = g_{\mu\nu}\delta_{n,-m}$, etc. $[g_{\mu\nu} = (1, -1, -1, -1)]$.

We now introduce some superconformal generators $G^{(0)}_r$, which satisfy the superconformal algebra

$$\{G^{(0)}_r, \ G^{(0)}_s\} = 2L^{(0)}_{r+s} + 6(r^2 - \frac{1}{4})\delta_{r,-s} \ ,$$

$$[L^{(0)}_n, \ G^{(0)}_r] = (\frac{n}{2} - r)G^{(0)}_{n+r} \ ,$$

$$[L^{(0)}_n, \ L^{(0)}_m] = (n-m)L^{(0)}_{n+m} + \frac{3}{2}n(n^2-1)\delta_{n,-m} \ . \tag{4}$$

Here

$$G^{(0)}_r = \sum_a (\tilde{J}^{(a)}\Psi^{(a)})_r = \frac{1}{\sqrt{7}}[\frac{1}{4i}(\tilde{\psi}\hat{\varphi}\psi)_r$$

$$+\frac{1}{4}(\tilde{\psi}\gamma_5\tau_i\hat{\kappa}_i\psi)_r + \frac{1}{4}((\tilde{\psi}\gamma_5\tau_i\psi)\vartheta_i)_r + \frac{1}{4}((\tilde{\psi}\tau_i\psi)\eta_i)_r$$

$$+\frac{1}{4}(\varsigma^{\mu\nu}(\tilde{\psi}\frac{1}{2}[\gamma_\mu, \gamma_\nu]\psi))_r + \frac{1}{i}((\psi\kappa_i)\vartheta_i)_r + \frac{1}{i}(\vartheta_i\vartheta_j\eta_k)_r\epsilon_{ijk}$$

$$+(\varsigma^{\mu\nu}\varphi_\mu\varphi_\nu)_r - \frac{1}{i}((\kappa_i\kappa_j)\eta_k)_r\epsilon_{ijk} - \frac{1}{i}(\eta_i\eta_j\eta_k)_r\epsilon_{ijk}$$

$$-(\varsigma^{\mu\nu}\kappa_{\mu i}\kappa_{\nu i})_r - (\varsigma^{\mu\nu}\varsigma_{\nu\lambda}\varsigma^\lambda_\mu)_r], \tag{5}$$

and $L^{(0)}_n$ are the ordinary fermion Virasoro operators, which correspond to a purely fermion action for the string (a kinetic term): $S = (i/8\pi\dot{a}')\int d\tau d\sigma(\tilde{\psi}\tilde{\partial}\psi) + \cdots$. Here

$$[L^{(0)}_n, \psi_r] = (-\frac{n}{2} - r)\varphi_{n+r} \ , \tag{6}$$

etc. It is interesting to note that the Sugawara form[4]

$$\sum_{a=V,A,S,P,T} J_l^{(a)} J_{n-l}^{(a)} = \frac{1}{2}\sum_l (\tilde{\psi}_{-l}\psi_{l+n})(l+\frac{n}{2}) = L_n^{(0)}(\psi)$$

gives us the part of $L_n^{(0)}(\psi)$ which corresponds to the ψ components.

In a sense, this algebra of operators $G_r^{(0)}$ is "empty," i.e., does not contain a dependence on the momentum of the state, P. In addition, after the integration over τ_i in (1), this algebra does not give us the P^2-pole contributions due to $1/L_0 - 1$. A nonempty result is found if we go over from the "unperturbed" operators $G_r^{(0)}$ to the deformed operators $G_r = G_r^{(0)} + (P\varphi)_r$, which satisfy the same superconformal algebra, (4), with

$$L_n = L_n^{(0)} + (PZ_n) - \delta_{n,0}\frac{P^2}{2} ,$$

$$Z_{n\mu} = \{G_r^{(0)}, \quad \varphi_{n-r,\mu}\}. \tag{7}$$

For the zero-dimensionality operator $KX(\tau) = \exp(iL_0\tau)kX(0)\exp(-iL_0\tau)$ it is now a simple matter to write the following representation:

$$kX(0) = \Lambda(k\tilde{X}_0 + \sum_{n\neq 0}\frac{\{G_r, (k\tilde{\varphi})_{n-r}\}}{in})\Lambda, \tag{8}$$

where the operator Λ projects onto the zeroth eigenvalue of the operator $Q = i(PZ)_0Q^+$ in L_0:

$$\Lambda = \lim_{R\to\infty}\frac{\int_{-R}^{R} d\alpha \exp(i\alpha Q)}{\int_{-R}^{R} d\alpha} . \tag{9}$$

The quantities $\tilde{\varphi}_r$ are the components of the "deformed" field $\tilde{\varphi}(\tau)$ which satisfy a condition analogous to (6):

$$[L_n, \tilde{\varphi}_r] = (-\frac{n}{2} - r)\tilde{\varphi}_{n+r} . \tag{10}$$

We can find $\tilde{\varphi}_r$ as a power series in the fermion fields from the equation

$$\tilde{\varphi}_r^{\lambda} = \varphi_r^{\lambda} - \sum_{m\neq 0}\frac{\{G_l^{(0)}, \tilde{\varphi}_{m-l}^{\mu}\}}{m}[\{G_s, \tilde{\varphi}_{-m-s,\mu}\}\tilde{\varphi}_r^{\lambda}]. \tag{11}$$

That condition (10) holds is obvious from the relation

$$[(P, Z_n), G_r^{(0)}] = n(P\varphi)_{n+r} \tag{12}$$

and from the superconformal algebra of the operators G_r [see (4)]. Here are the first few terms in the expansion of $\tilde{\varphi}_r$:

$$\tilde{\varphi}_r^{\lambda} = \varphi_r^{\lambda} - \frac{1}{\sqrt{7}}\sum_{m\neq 0}\frac{Z_{m\mu}}{m}(\varsigma^{\mu\lambda})_{r-m} + \cdots .$$

We are left with the problem of formulating an operator expression for the vertex for the emission of a π meson with the correct quantum numbers and the necessary superconformal properties. As in the Neveu–Schwarz model,[5] the vertex operator which we need is given by the anticommutator with the supergenerator G_r:

$$V(0) = \Lambda\{G_r, k\kappa_i(0)\exp(ikX(0))\}\Lambda. \tag{13}$$

The operator Λ does not distort the superconformal properties of the vertex, since the operator $Q = iPZ_0$ commutes with all the gauge operators $G_r L_n$, L_0. A standard analysis for superconformal string amplitudes with the help of vertices (13) reveals that the physical spectrum of states has no tachyon. In addition, the residue of the massless vector states also vanishes in the sum of amplitudes (1). In contrast with the ordinary spectrum of (dual) string models, this spectrum of physical states of the new string has no particles with a high spin on the main trajectory or on any of the daughter trajectories, by virtue of the Pauli principle for the components of the fields ψ, φ, κ, and ζ. With increasing index of the daughter trajectory, this divergence of the trajectories occurs at progressively higher spins. The maximum spin increases asymptotically in proportion to the mass of the states. The physical spectrum of this new model will be analyzed further in subsequent papers.

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