Superradiance of an electron swarm in a periodic magnetic field

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(Submitted 19 August 1991)
Pis'ma Zh. Eksp. Teor. Fiz. **54**, No. 8, 445–448 (25 October 1991)

A spontaneous coherent emission (superradiance) occurs as an electron swarm (or slab) moves through a periodic magnetic field. The coherence stems from a bunching of some of the particles inside the swarm because of a ponderomotive force: a radiation reaction force.

1. The emission of radiation by an electron swarm moving through a periodic magnetic field (an undulator field) can serve as an example of spontaneous coherent emission (or Dicke superradiance[1–4]) in systems of classical oscillators. It is assumed that the size of the swarm is large at the scale of the wavelength of the light but small at the scale of the length of the undulator, so the lifetime of an electron in the undulator field can be assumed infinite (in contrast with the situation which is customarily studied in the theory of free-electron lasers[5,6]). This swarm constitutes a moving active resonator in which an instability[7] involving no threshold occurs and leads to a bunching of the particles and a subsequent coherent emission by them. While the frequency of the emission in the comoving frame of reference, K', will be the same in the various directions and will be approximately the same as the electron oscillation frequency, the frequency in the laboratory frame of reference, K, will depend on the observation angle, because of the Doppler effect. This emission thus simultaneously exhibits the properties of induced processes (a coherence) and spontaneous processes (anisotropy, multiplicity of frequencies, and absence of a threshold).

2. Here we wish to examine the nonlinear stage of the superradiance effect on the basis of a $1D$ model. We assume that the swarm is a slab of width b along the z direction, while it is infinite along the x and y directions. We analyze the situation in the comoving frame K', in which the undulator field transforms into the field of an electromagnetic pump wave given by the vector potential

$$\vec{A}_u{}' = \mathrm{Re}[\vec{x}_0 A'_u e^{i(\omega'_u t' + h'_u z')}], \tag{1}$$

where $h'_u = \gamma h_u$, $\omega'_u = \gamma \cosh_u$, $h_u = 2\pi/d$, d is the period of the undulator, $\gamma = (1 - v_0^2/c^2)^{-1/2}$, and v_0 is the translational velocity of the slab. The pumping imparts an oscillatory velocity to the particles: $v'_x = \mathrm{Re}\left[(eA'_u/mc)e^{i(\omega'_u t + h'_u z')}\right]$. The field emitted (or scattered) by the oscillating particles can be written as two waves, traveling in the $\pm z'$ directions:

$$\vec{A}'^{\pm} = \mathrm{Re}[\vec{x}_0 A'^{\pm}_s(z', t')e^{i(\omega'_s t' \mp h'_s z')}]. \tag{2}$$

Here $h'_s = \omega'_s/c$, and ω'_s is the carrier frequency (below, we set $\omega'_s = \omega'_u$). The combined effects of fields (1) and (2) on the electrons gives rise to an average ponderomotive force, which is responsible for a bunching of the electrons:

$$F'_{\rm pond} = -\frac{e^2}{4m\omega'^2_u}\frac{\partial}{\partial z'}{\rm Re}[A'_u A'^+_s e^{i(h'_s+h'_u)z'} + A'_u A'^-_s e^{i(h'_s-h'_u)z'}]. \tag{3}$$

For a numerical simulation of the superradiance process, we partition the slab into N_Σ planes ("macroelectrons") at the coordinates $z'_n(t'_\perp z'_{0n})$, where z'_{0n} are the initial coordinates of the macroelectrons. These macroelectrons interact with each other via the ponderomotive force in (3) and the repulsive Coulomb force $F'_{\rm coul}$ (we are assuming that the static charge of the electrons is neutralized by a fixed ion background). In Lagrangian variables, the equations of motion of these macroelectrons can be written

$$\frac{d\beta'_n}{d\tau'} = F'^n_{\rm pond} + F'^n_{\rm Coul}, \quad \frac{dZ'_n}{d\tau'} = \beta'_n, \tag{4}$$

where

$$F'^n_{\rm pond} = \frac{\alpha'^2_u q' B'}{8N_\Sigma}\left[-\sum_m^{N^+(n)} \nu_+ \cos(Z'_n - Z'_m) + \sum_m^{N^-(n)} \nu_- \cos(Z'_n - Z'_m)\right],$$

$$F'^n_{\rm Coul} = \frac{q'B'}{2N_\Sigma}[N^+(n) - N^-(n) - N^+_i(n) + N^-_i(n)].$$

We specify the initial conditions to be

$$Z'_n|_{\tau'=0} = B'\left(\frac{n-1}{N_\Sigma - 1} - \frac{1}{2}\right), \quad \beta'_n|_{\tau=0} = 0, \quad n = [1, N_\Sigma].$$

Here $\tau' = \omega'_u t'$; $Z' = \omega'_u/cz'$; $v' = \beta'_c$ is the velocity of longitudinal displacements; $q' = \omega'^2_p/\omega'^2_u$; $\omega'_p = \sqrt{4\pi e^2\rho'_0/m}$; ρ'_0 is the density of the slab;

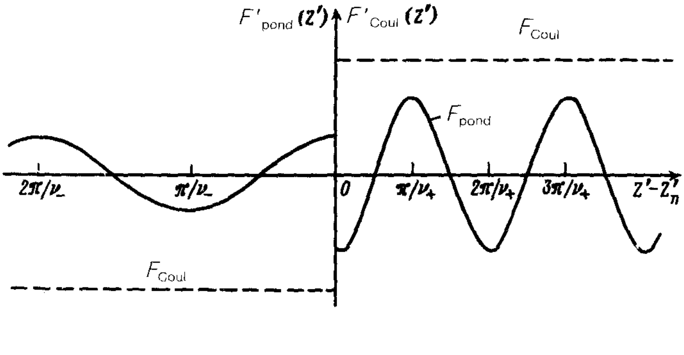

FIG. 1. Profile along the longitudinal coordinate of the ponderomotive force which one macroelectron exerts on the others. The dashed lines show the repulsive Coulomb force.

$\nu_{\pm} = (h'_s \pm n'_u)c/\omega'_u$; $N^{\pm}(n)$ is the number of electrons with coordinates larger than (smaller than) Z'_n; and $N_i^{\pm}(n)$ is the same number, for the neutralizing ion background.

The amplitudes of the waves emitted by the electrons are given by

$$\alpha'^{\pm}_s = eA'^{\pm}_s/mc^2 = \frac{iq'\alpha'_u B'}{4N_\Sigma}\sum_m^{N^{\pm}} e^{-i\nu_{\pm}|Z'-Z'_m(\tau')|}.$$

Figure 1 shows the profile along the longitudinal coordinate of the ponderomotive force which one macroelectron exerts on the others. In contrast with the repulsive Coulomb force (shown by the dashed lines), this force is attractive in the near zone. This behavior of the interaction force should cause an instability and a breakup of the slab into coherently emitting swarms. For the initial stage of the process ($\tau' \simeq 10$–20), this effect is clear from Fig. 2, which shows the time evolution of the coordinates of the electrons. For the relativistic ($\gamma \gg 1$) motion of the slab (in the laboratory frame K) with which we are dealing here, with $h'_s \simeq h'_u$ and $\nu^+ \gg \nu^-$ ($\nu^+ = 2$, $\nu^- = \gamma^{-2}/2$), the motion of the electrons is governed by that component of the ponderomotive force which stems from the wave A'^{+}_s: the wave propagating opposite the pump wave (in

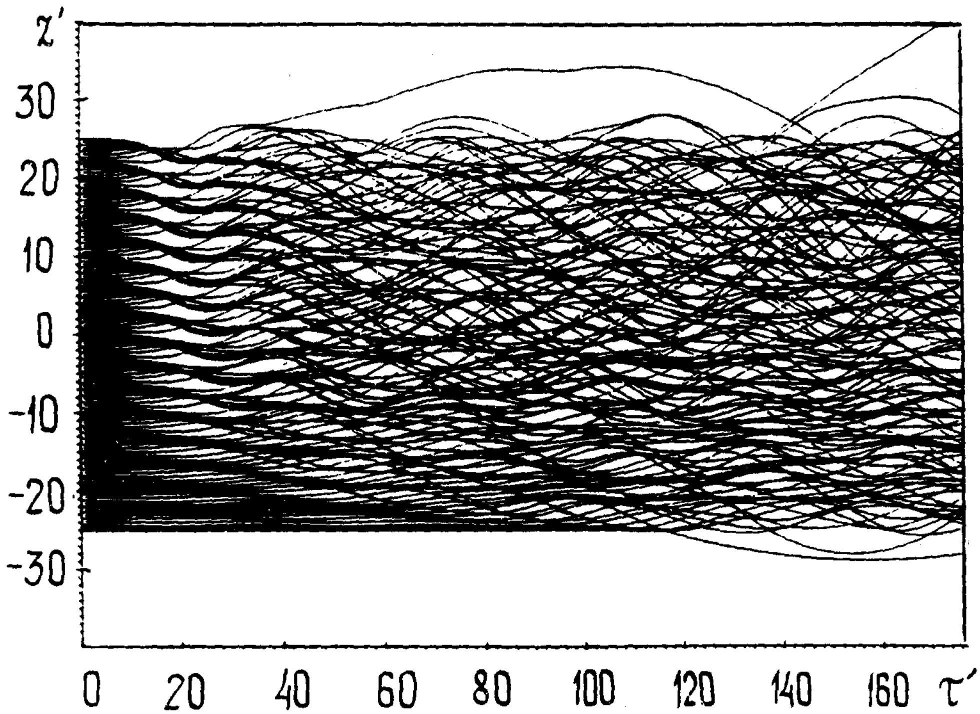

FIG. 2. Time evolution of the coordinates of the macroelectrons ($q' = 0.04$, $\alpha'_u = 0.77$, $\nu_+ = 2$, $\nu_- = 0.1$, $B' = 50$).

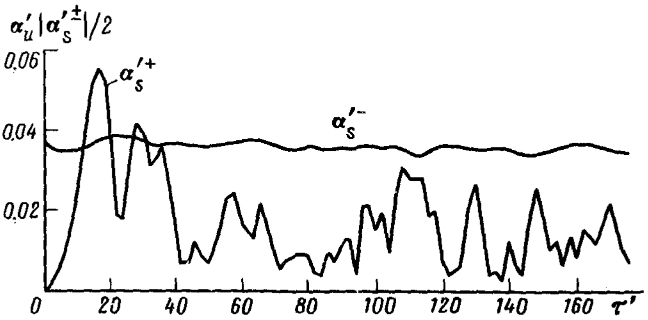

FIG. 3. Time evolution of the amplitudes $|\alpha_s'^{\pm}|$ of the emitted waves ($q' = 0.04, \alpha_u' = 0.77, \nu_+ = 2, \nu_- = 0.1, B' = 50$).

the direction of the translational motion in frame K). Correspondingly, the electrons become bunched in such a way that they make the amplitude of this wave substantially larger than that at the initial time (Fig. 3). In the case of an ideal bunching of the electrons, the maximum amplitude of this wave is determined by $|\alpha_s'^+| = q'\alpha_u' B'/4$. The peak field amplitude in Fig. 3 is smaller than this value by a factor of about 2.7. At certain time $\tau' \simeq 15$, the emission of the particles thus reaches a high degree of coherence. After a long time, the particles in the slab undergo a pronounced mixing, and the wave amplitude $A_s'^+$ falls off.

3. We turn now to some particular features of the superradiance in the laboratory frame of reference. While the frequencies of the waves emitted by the slab in the $\pm z'$ directions are the same in the comoving frame of reference, in the laboratory frame these frequencies become quite different as $v_0 \to c$: $\omega^+/\omega^- \simeq 2\gamma^2 \gg 1$. Correspondingly, when we take in account the conservation of the number of photons, we conclude that the power radiated along the positive z direction is much higher than that radiated in the opposite direction:

$$\frac{P^+}{P^-} = \frac{\omega^+}{\omega^-}\frac{P'^+}{P'^-} \simeq 2\gamma^2,$$

where $P'^{\pm} = |A_s'^{\pm}|/8\pi c\omega_u'^2$. In the laboratory frame, most of the energy of the superradiance is thus concentrated in the short-wave component. The effect discussed here may thus be regarded as a promising method for producing coherent emission, particularly in ranges in which no efficient reflectors are available.

The analysis above also applies to the case in which oscillations are imparted to a moving or nonmoving electron swarm by a traveling electromagnetic pump wave.

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Translated by D. Parsons