Pinning of solitons by Abrikosov vortices in distributed Josephson junctions

L. G. Aslamazov and E. V. Gurovich
Institute of Steel and Alloys

(Submitted 24 May 1984)
Pis'ma Zh. Eksp. Teor. Fiz. **40**, No. 1, 22–25 (10 July 1984)

In a distributed Josephson junction the presence of an Abrikosov vortex in the superconducting film leads to a new type of perturbation of the sine-Gordon equation. This inhomogeneity makes it possible to distinguish the polarity of solitons and leads to several physical effects that can be observed experimentally.

There has recently been great interest in the study of distributed Josephson junctions (DJJ), in which magnetic-flux quanta (solitons) can propagate. Microscopic circuits[1] and microscopic resistances[2] are used to pin solitons in such junctions. In this letter we examine the effect of fluxoids, which are localized in the superconductor [for example, Abrikosov vortices (AV) pinned by artificially created inhomogeneities], on moving solitons. This method of controlling solitons has a number of advantages: The polarity of the solitons can be distinguished and it is easy to create and annihilate the controlling centers.

The DJJ is one of many physical systems that are described very well by the sine-Gordon equation (SG), in which the inhomogeneity of the contact leads to the appearance of an additional term. In the presence of AV the equation for the difference in the phases of the order parameter φ in the superconductors has the form

$$\varphi_{tt} - \varphi_{xx} + \sin\varphi = -\alpha\varphi_t - \gamma - \sum_i \eta_i \delta_x (a_i - x)$$
$$\eta_i = 2\pi e^{-\kappa b_i}, \tag{1}$$

where the coordinate x, along which the solitons propagate, is measured in units of the Josephson penetration depth λ_J, the time t is measured in units of the inverse Josephson plasma frequency, and the indices x and t denote differentiation with respect to the corresponding variables.

The first term on the right side describes dissipation related to the passage of a normal current across the junction, the second term describes the uniformly distributed current (bias current), and the third term corresponds to the perturbation created by the AV in the superconductor situated at points with the coordinates a_i at distances b_i from the junction plane (the axes of the vortices are perpendicular to the x axis and parallel to the junction plane); the parameter $\kappa = (\lambda_J/\lambda_L) \gg 1$, where λ_L is the London penetration depth.

The perturbation in the SG equation, caused by the AV, is due to surface currents, which lead to an additional gradient of the phase in the superconductor.[3] These currents are localized in the region $\Delta x \sim \lambda_L$, near the vortex, which is much smaller than the characteristic scale of the change in the phase λ_J of the soliton. The distribution of surface currents can therefore be modelled by a δ-function, and its derivative appears in the equation for the phase. This is the first time a modified SG equation of this kind has been studied.

The physical meaning of the perturbation becomes clear if Eq. (1) is written as a Hamiltonian. The Hamiltonian will then have an additional term

$$\mathcal{H}^{P}(\varphi) = \int \eta\delta(a-x)\varphi_x dx = 2\pi\varphi_x(a)e^{-\kappa b}, \tag{2}$$

which in the appropriate units coincides with the energy of the AV in the magnetic field of the soliton in the London approximation.

To study the dynamics of a single soliton in DJJ in the presence of AV we will use the perturbation theory developed in Ref. 4. For the coordinates of the center of the soliton $Z(t)$ and its velocity $u(t)$ we find from Eq. (1) an autonomous system of ordinary differential equations.

$$\begin{aligned} X_t &= u + \frac{\eta u}{4}\operatorname{sch}(X/\sqrt{1-u^2})\left[1 - \frac{X\tanh(X/\sqrt{1-u^2})}{\sqrt{1-u^2}}\right] \\ u_t &= \frac{\eta}{4}\sqrt{1-u^2}\tanh(X/\sqrt{1-u^2})\operatorname{sch}(X/\sqrt{1-u^2}) \\ &\quad + \frac{\pi\gamma}{4}(1-u^2)^{3/2} - \alpha u(1-u^2), \end{aligned} \tag{3}$$

where the sign of η is determined by the polarity of the soliton with respect to AV ($\eta > 0$ for identical polarities), and the sign of γ is determined by the orientation of the bias current.

In the simplest case of no dissipation and no bias current ($\alpha = \beta = 0$) Eq. (3) can be integrated completely. The infinite trajectories in the phase plane (u, X) are given by

$$X(u) = \pm\sqrt{1-u^2}\,\operatorname{Ars\,cosh}\left(\frac{4}{\eta}\left[\left(\frac{1-u^2}{1-u_\infty^2}\right)^{1/2} - 1\right]\right), \tag{4}$$

where u_∞ is the velocity of the soliton at infinity. If the soliton is attracted to the AV ($\eta < 0$), there will also be limiting cycles for which we give the relation between the velocity in the equilibrium position $u_{\max}$ and the maximum deviation $X_{\max}$ from the equilibrium position:

$$u_{\max}^2 = \frac{\eta}{4}(1 - \operatorname{sch} X_{\max}). \tag{5}$$

We see from Eqs. (4) and (5) that three types of motion are possible, depending on the polarity of the soliton and its initial velocity: reflection of the φ^+ soliton from the vortex ($\eta > 0$, $u_\infty < \sqrt{\eta/2}$, oscillations of the φ^- soliton near the vortex ($\eta < 0$, $u_\infty = 0$) and departure of solitons along the side away from the vortex (in the remaining cases).

In the presence of dissipation and bias current, Eq. (3) can be solved numerically on a computer. Here again we see a considerable difference in the dynamics of solitons with different polarity. Figure 1 shows the different phase trajectories for the φ^+

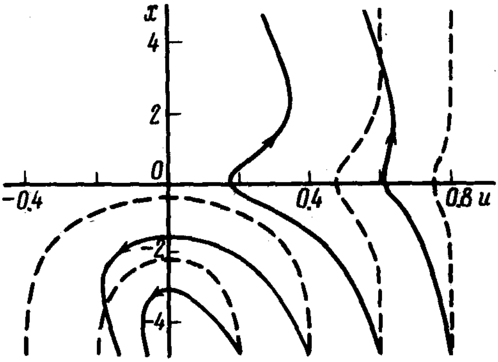

FIG. 1. Phase trajectory of the motion of a φ^+ soliton with $\gamma = 0$; $\alpha = 0$; $\eta = 0.4$ (dashed curve) and with $\gamma = 0$; $\alpha = 0.05$; $\eta = 0.4$ (solid curve).

soliton, and Fig. 2 shows the phase trajectories for the φ^- soliton. As is evident, the dissipation leads to pinning of the φ^- soliton, whereas the motion of the φ^+ soliton does not change substantially. The φ^+ soliton can also be pinned in the presence of the bias current. However, the pinning point in this case is located far from the AV (for small values of γ). The coordinates X_0 of the pinning points can generally be determined from the equation

$$\frac{\pi\gamma}{\eta} + (\tanh X_0)(\operatorname{sch} X_0) = 0. \tag{6}$$

The critical value of the bias current, at which the pinning of the solitons becomes impossible, is $\gamma_c = \eta/2\pi$.

In the harmonic approximation, it is possible to determine the oscillation frequencies near the position of equilibrium,

$$\Omega^2 = \frac{\eta}{4}\operatorname{sch} X_0 (\tanh^2 X_0 - \operatorname{sch}^2 X_0). \tag{7}$$

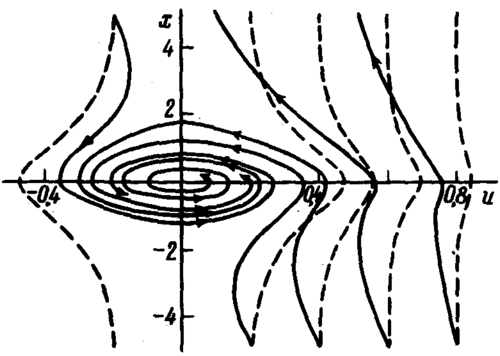

FIG. 2. Phase trajectory of the motion of a φ^- soliton with $\gamma = 0$; $\alpha = 0$; $\eta = 0.4$ (dashed curve) and with $\gamma = 0$; $\alpha = 0.05$; $\eta = 0.4$ (solid curve).

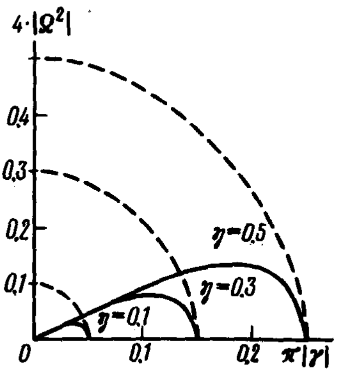

FIG. 3. Dependence of the square of the oscillation frequency Ω^2 of the φ^- soliton (dashed curve) and of the φ^+ soliton (solid curve) on the bias current γ for different values of η.

Depending on the sign of η, there are two substantially different branches of the spectrum, which correspond to oscillations of the φ^+ and φ^- solitons (Fig. 3).

A pulse of bias current, which accounts for the initial velocity of the soliton, can be used to pump the oscillations. In this case the maximum deviation from the position of equilibrium in the absence of the bias current can be calculated from Eq. (5). We also note that the AV can split breezers that propagate in the DJJ.

All of the effects described above can be observed experimentally in DJJ of finite length by using techniques that permit recording the reflection of a soliton from the junction ends,[5] fixing the time at which the soliton passes through the AV by using two sensors situated on both sides of the AV (Ref. 6), or directly analyzing the output signal on a computer.[7] The change in the polarity of the soliton, which usually accompanies reflection from the end of the line, must be taken into account in this case.

The AV can be pinned with the help of the local decrease of the order parameter, for example, by depositing on the superconducting film at certain locations a layer of normal metal (making use of the proximity effect). If the film is reduced in thickness locally and a current is passed through it, then the increases in the current density at this location perturbs the current that passes through the junction. This process is also described by Eq. 1.

The authors thank A. A. Abrikosov, V. N. Gubankov, V. A. Koshel'ts, A. I. Larkin, and A. T. Filippov for valuable discussions of the results.

[1]Solitony v Deĭstvii (Solitons in Action), Mir, Moscow, 1981.
[2]Yu. S. Gal'pern and A. T. Filippov, Pis'ma Zh. Eksp. Teor. Fiz. **35**, 470 (1982) [JETP Lett. **35**, 580 (1982)].
[3]Yu. P. Denisov, Fiz. Tverd. Tela (Leningrad) **18**, 119 (1979) [Sov. Phys. Solid State **18**, 66 (1979)].
[4]D. W. McLaughlin and A. C. Scott, Appl. Phys. Lett. **30**, 545 (1977).
[5]B. Dueholm, O. A. Levring, J. Mygind, N. F. Pedersen, O. H. Soerensen, and M. Cirillo, Phys. Rev. Lett. **46**, 1299 (1981).
[6]A. C. Scott, F. Y. F. Chu, and S. A. Reble, J. Appl. Phys. **47**, 3242 (1976).
[7]A. Matsuda and T. Kawakami, Phys. Rev. Lett. **51**, 694 (1983).

Translated by M. E. Alferieff
Edited by S. J. Amoretty