Spin fluctuations in disordered systems near the metal–insulator transition

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A disordered system of interacting electrons is analyzed by the renormalization-group method. It is shown that the metal–insulator transition occurs against the background of a spontaneous change in the spin density.

1. Attempts have recently been made to describe the metal–insulator transition in a disordered system by the renormalization-group (RG) method as in the theory of second-order phase transitions. For free electrons this approach was realized in Refs. 1–3. The interaction of diffusing electrons is very strong,[2] and it greatly complicates the problem because of frequency mixing. The author constructed a scheme for calculating the Coulomb interaction in the RG equations,[3] which permitted examining[4,5] the behavior of the system in the presence of magnetic impurities and in the presence of a magnetic field, when the Zeeman splitting is large. In these cases, the resistance of the film ($d = 2$) becomes infinite as a result of renormalization, and for $d > 2$ the RG equations have an unstable fixed point which corresponds to the metal–insulator transition.

The renormalization-group equations ($d = 2$) have recently been derived[6] in the absence of magnetic interactions, when the scattering by impurities is a purely potential scattering (both the Coulomb correlations and the contribution from cooperons were taken into account). These equations were derived working from the scheme suggested by Finkel'shtein.[3] We encountered a qualitatively new situation: When a certain scale is reached, the constants describing the interaction of electrons diverge (logarithmic pole), whereas the resistance remains finite. In this case, there is a tendency for the spin density to change: As the pole is approached, the spin susceptibility diverges, while the coefficient of spin diffusion approaches zero. The difference from the situations examined in Refs. 4 and 5 arose due to the spin diffusion modes, rather than due to the inclusion of cooperons. Since these modes are completely suppressed by spin scattering and partially suppressed by Zeeman splitting,[7] the electron system turned out to be "prepared" for localization in the presence of magnetic interactions. If, on the other hand, there are no external actions, then the localization occurs against the background of a spontaneous change in the spin density. In this paper we discuss the role of spin fluctuations for $d = 2 + \epsilon (\epsilon > 0)$.

2. The disordered system is described by the following set of quantities: the constant density of states ν; the diffusion coefficient D; the parameter z, which takes into account the renormalization of the frequency[3,4] and the amplitudes $(2\Gamma - \Gamma_2)$, Γ_2, and Γ_C, which describe the interaction of electrons. The quantity $(2\Gamma - \Gamma_2)$ describes the interaction of fluctuations of the density of particles, Γ_2 describes the spin density, and Γ_C takes into account the interaction of cooperons. The Fermi-liquid corrections and the static screening are incorporated into these amplitudes (the case of the interaction Coulomb, i.e., the long-range, interaction is examined). With renormalization, the relation $2\nu\Gamma - \nu\Gamma_2 = z$, is satisfied.[3] This situation corresponds to the condition of incompressibility of the electron liquid.

In lowest order with respect to ϵ, we can write the renormalization-group equations as follows[6]:

$$dg/d\xi = -\epsilon g + 2g^2\left[5 - 3\frac{1+w}{w}\ln(1+w)\right]; \qquad dw/d\xi = g(1+w)^2 \tag{1}$$

$$d\ln z/d\xi = (-1+3w)/(1+w)^2 \tag{2}$$

where $w = \nu\Gamma_2/z$; $\xi = \ln(\lambda_0/\lambda)$ (λ is the cutoff momentum) and $g = \lambda^{d-2}/(2\pi)^2\nu D$ is a dimensionless parameter proportional to the resistance of a specimen with dimensions $L \sim \lambda^{-1}$. In (1) and (2) the contribution of the amplitude Γ_C has been dropped, since it is unimportant in the lowest-order approximation with respect to ϵ.[6]

In Ref. 6 the correlation function of the spin density $\chi(q,\omega) = \chi_S D_S q^2/(D_S q^2 - i\omega)$ was calculated and the spin susceptibility χ_S and the coefficient of spin diffusion D_S were found[1)]

$$\chi_S = {}^1/_2\,(g_L\mu_B)^2\,\nu z(1+w); \quad D_S = D/z(1+w). \tag{3}$$

Taking (2) into account, we find

$$\chi_S = {}^1/_2\,(g_L\mu_B)^2\,\nu(1+\nu\Gamma_2^0)f(w)/f(w_0); \quad f(w) = (1+w)^4\exp[-4w/(1+w)],$$

where $w_0 = \nu\Gamma_2^0$ is the seed value of $\nu\Gamma_2$. In this approximation χ_S depends only on w, which is determined by Eqs. (1). The trajectories $g(w)$ of Eqs. (1) are shown schematically in Fig. 1. For small values of g_0 (g_0 is the initial value of the parameter g), the trajectories $g(w)$ reach $g = 0$ for finite values of w. However, for values of g_0, which are higher than a certain value of g_m, the trajectories change: For such trajectories we have $w \to \infty$, so that $g \neq 0$. An important point there is that at $g_0 > g_m$ the divergence of w (and therefore of χ_S) occurs for a finite value $\xi = \xi_{cr}(g_0, \nu\Gamma_2^0)$.

3. Thus, if the seed conductivity is low ($g_0 > g_m$), then when a scale ξ_{cr} is reached, there is a tendency for the spin density to change: $\chi_S \to \infty$, while $D_S \to 0$, although the conductivity $\sigma = 2e^2\nu D$ remains finite. What is the further development at $\xi > \xi_{cr}$? Because of the fluctuations of the composition, the sample contains regions where the instability develops earlier and islands of localized spin density appear. These localized moments magnetize the remaining electrons. Since the decrease in magnetization with

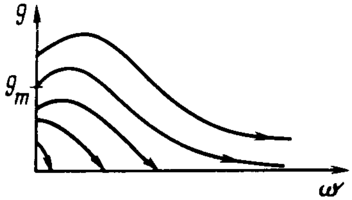

FIG. 1.

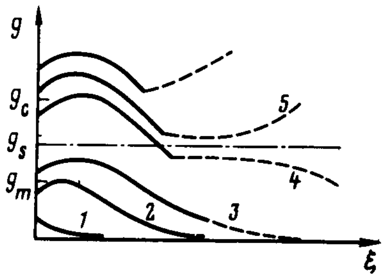

FIG. 2.

the distance r is determined by the same processes that lead to the renormalization of χ_S, we find $\chi_S \sim (1/r^d)\, g(r) f(w) w(r)$ (in the case of a good metal, when $g(r) \sim r^{2-d}$, this value was obtained in Ref. 9). The slow decrease in magnetization suppresses the long-wavelength spin-diffusion (and cooperon) modes because of the separation of the Fermi surface of electrons with different spin projections, and the development of the instability stops. With an impurity concentration of $n \sim n_c$ (n_c is the concentration at the transition point; n_c corresponds to $g_0 = g_c$) the direct exchange,[10] which leads to antiferromagnetic interaction of the angular momenta, is important. As a result, the ferromagnetic interaction resulting from the indirect exchange will be blocked.[9] The subsequent ($\xi > \xi_{cr}$) behavior of $g(\xi)$ is therefore described by the equation

$$dg/d\xi = -\epsilon g + 2g^2, \tag{5}$$

which was derived for the magnetic impurities. Equation (5) has a fixed point $g = g_S$, which corresponds to the metal–insulator transition. We finally obtain the following picture: Near the transition, the Coulomb correlations lead to the formation of a random spin density wave, against the background of which the metal–insulator transition occurs.

4. Figure 2 shows schematically the behavior of $g(\xi)$ for different values of g_0. The solid lines show the solutions of (1) up to $\xi \approx \xi_{cr}$ and the dashed lines represent the solutions of (5). The dot-dashed curve shows the position of g_S. The transition occurs at $g_0 = g_c$ such that the condition $g(\xi_{cr}) = g_S$ is satisfied when ξ_{cr} is reached: Curves 4 and 5 describe the metal and the insulator, respectively, very close to the transition. Curves 1–3 illustrate the behavior of the spin susceptibility. At $g_0 \ll g_m$ (curve 1) χ_S has a purely Pauli character; at $g_0 < g_m$ (curve 2) χ_S differs from the Pauli curve due to the increase in w; at $g_m < g_0 < g_c$ (curve 3) χ_S contains a Curie-type contribution due to the formation of localized moments.

Measurements have shown[11] that in the metallic phase in the region $n_c < n \lesssim 2n_c$ χ_S does not satisfy the Pauli law and has an appreciable Curie component.

The picture described above was based on the fact that the amplitude Γ_2 increases most strongly with renormalization. Since $(2\Gamma - \Gamma_2)$, which describes the behavior of the charge density, also diverges in the limit $\xi \to \xi_{cr}$ (although more slowly than Γ_2), another type of transition can also occur. We assume that at $g_0 \approx g_m$, the charge density changes simultaneously with the change in the spins. The conductivity in this case vanishes due to pinning. Since the trajectory which emerges from $g_0 = g_m$ corresponds to finite conductivity, σ vanishes almost discontinuously near the transition in this case.

It was found in Si:P[12] that for $(n-n_c)/n_c \sim 1\%$ the sign of the temperature correction to σ changes. In explaining this fact it was assumed[12] that the screening radius r_{scr} diverges near the transition. It was later found[3,13] that since the compressibility $\partial N/\partial\mu$, which determines the quantity r_{scr}, does not contain large diffusion corrections, this explanation is not valid. The author assumed that the sign of $d\sigma/dT^{1/2}$ changes because the change occurring here near the transition is similar to the one described above. To resolve this problem we need data on the spin susceptibility of these samples. Unfortunately, such data are not available at this time.

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[1)]Equations (3) were obtained independently in Ref. 8, where the model of Ref. 3 was discussed again for the Landé factor $g_L = 0$. In this case the magnetic field acts only on the cooperons and suppresses them. At $d = 2$ χ_S in this case diverges as a result of renormalization (see the Appendix in Refs. 6 and 8).

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