Anomalous viscosity and rotation of a tokamak plasma

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An expression is derived for the viscosity coefficient due to plasma microturbulence. The corresponding drag, which is proportional to the energy lifetime, can explain the anomalously rapid decay of plasma rotation in a tokamak.

During the oblique injection of particles into a tokamak plasma, the plasma column is observed to go into rotation in the toroidal direction.[1,2] When the particle injection ends, the rotation velocity rapidly decays. The decay time cannot be explained by the neoclassical theory. At present, the most likely mechanism for the decay of the rotation seems to be friction with the neutral gas. Again in this case, however, the theoretical decay time is several times longer than the experimental time. Furthermore, this mechanism runs into difficulty with the experimental fact that the decay time is on the order of the energy lifetime of the plasma.

In the present letter we wish to propose a possible slowing mechanism which involves the anomalous viscosity of a tokamak plasma. This viscosity emerges in a natural way from the system of equations recently proposed in Ref. 3 for a self-consistent description of transport processes in a tokamak. The expresion derived here for the slowing time is proportional to the energy lifetime.

The system of equations of Ref. 3 consists of an equation for a generalized vortex,

$$\rho_0 \frac{\partial \Gamma}{\partial t} + \frac{\rho_0 c}{B_0} [\mathbf{e}_z , \nabla \varphi] \cdot \nabla \Gamma = \frac{(\mathbf{B}, \nabla)}{4\pi} \Delta_\perp \psi , \qquad (1)$$

where

$$\Gamma = \frac{c}{B_0} \Delta_\perp \varphi = \mathrm{rot}_z \mathbf{v}_\perp , \quad \mathbf{B} = \mathbf{B}_0 + [\mathbf{e}_z , \nabla \psi], \quad \Delta_\perp \psi = \frac{4\pi}{c} j_\parallel ,$$

and ρ_0 is the plasma density; the continuity equation,

$$\frac{\partial n}{\partial t} + \frac{c}{B_0} [\mathbf{e}_z , \nabla \varphi] \cdot \nabla n = \frac{c}{4\pi e} \frac{(\mathbf{B}, \nabla)}{B_0} \Delta_\perp \psi ; \qquad (2)$$

and Ohm's law,

$$\frac{\partial \psi}{\partial t} = c \frac{\mathbf{B}}{B_0} \cdot \left(\nabla \varphi - \frac{\nabla p_e}{en} \right) + \frac{c^2 \hat{\eta}}{4\pi} \Delta_\perp \psi , \qquad (3)$$

where the operator $\hat{\eta} = \hat{\sigma}^{-1}$ represent the resistivity of the plasma, and the operator $\hat{\sigma}$ represents the conductivity.[3] Equations (1)–(3) could be supplemented with an equation for T_e, but we will not need it here. Although only the velocity component $\mathbf{v}_\perp$ appears in these equations, they can be used to describe the toroidal rotation. The poloidal rotation is slowed in a time $\tau_p \sim (\langle |\nabla \psi|^2 \rangle \rho_0 / \mu \langle (\mathbf{B} \cdot \nabla B / B_0)^2 \rangle)$, which is considerably shorter than the time $(\tau_\varphi$ over which the toroidal rotation is slowed[4] (μ is the neoclassical viscosity coefficient, and the angle brackets denote an average over a magnetic surface). At times $t \gg \tau_p$, the poloidal components $\mathbf{v}_\parallel$ and $\mathbf{v}_\perp$ cancel out to within terms proportional to ∇T, and Eqs. (1)–(3) can be used to describe the toroidal rotation.

The term $(\mathbf{B}, \nabla) j_\parallel$ in Eq. (2) was shown in Ref. 3 to give rise to a plasma diffusion across the magnetic surfaces with a diffusion coefficient given in order of magnitude by $D \sim (c^2 v_e / \omega^2_{pe} qR) \varepsilon^\alpha$ $(1 \leqslant \alpha \leqslant 2)$, when the finite conductivity is taken into account. We note that exactly the same term, $(\mathbf{B}, \nabla) \Delta_\perp \psi$, appears on the right side of vortex equation (1). The case of no average macroscopic rotation velocity was studied in Ref. 3. When there is such a velocity, a vorticity flux proportional to the vorticity gradient appears on the right sides of (1) and (2). We will demonstrate this point for the simple case of a plasma of homogeneous density and temperature (when gradients of n and T

are taken into account, terms proportional to ∇n and ∇T are incorporated in the fluxes in a simple additive manner; see Ref. (3). As in Ref. 3, we introduce the toroidal surfaces $\Phi = \text{const}$, which move with the plasma:

$$\frac{\partial \Phi}{\partial t} + \frac{c}{B_0}[\mathbf{e}_z, \nabla\varphi]\cdot\nabla\Phi = 0. \tag{4}$$

We initially place magnetic lines of force on these toroidal surfaces: $\mathbf{B}\cdot\nabla\Phi = 0$ $(t = 0)$. At subsequent times we then have, as was shown in Ref. 3,

$$\frac{d}{dt}(\mathbf{B}, \nabla\Phi) = [\nabla\hat{\eta}\Delta_\perp\psi, \nabla\Phi]_z\ ; \quad \frac{d}{dt} = \frac{\partial}{\partial t} + \frac{c}{B_0}[\mathbf{e}_z, \nabla\varphi]\cdot\nabla\ . \tag{5}$$

We can now calculate the change in the vorticity in the volume bounded by a surface $\Phi = \text{const}$:

$$\frac{\partial}{\partial t}\int_{\Phi=\text{const}}\Gamma\, dV = \int\frac{\partial\Gamma}{\partial t}dV + \int\Gamma\,\mathbf{v}\cdot d\mathbf{s} = \int(\mathbf{B},\nabla)\Delta_\perp\psi\frac{dV}{4\pi\rho_0} = \frac{1}{4\pi\rho_0}\int\Delta_\perp\psi\,\mathbf{B}\cdot d\mathbf{s}. \tag{6}$$

In the differentiation in (6) we made use of the circumstance that the surface $\Phi = \text{const}$ itself undergoes a displacement over time in accordance with (4). The integration in the last integral in (6) is carried out over a surface $\Phi = \text{const}$. Using $d\mathbf{s} = (\nabla\Phi/|\nabla\Phi|)ds$ and (5), we can rewrite (6) as follows:

$$\frac{\partial}{\partial t}\int\Gamma\, dV = -\int\Pi\, ds, \tag{7}$$

where Π is the vorticity flux out of the volume bounded by the surface $\Phi = \text{const}$:

$$\Pi = \frac{1}{4\pi\rho_0}\int_{-\infty}^{t}\Delta_\perp\psi(t)\frac{[\nabla(\hat{\eta}\Delta_\perp\psi(t')),\ \nabla\varphi(t')]_z}{|\nabla\Phi|}dt'. \tag{8}$$

For small fluctations and nearly cylindrical average surfaces, this flux has only a radial component and is given by

$$\Pi = \frac{1}{4\pi\rho_0}\int_{-\infty}^{t} j_\parallel(t)\frac{\partial}{r\partial\theta}\tilde{E}_\parallel(t')dt'. \tag{9}$$

We have rewritten Π in terms of more-graphic quantities: the current $\tilde{j}_\perp = (c/4\pi)\Delta_\perp\psi$ and the electric field $\tilde{E}_\parallel = \hat{\eta}\tilde{j}_\parallel$. In a calculation from (9) we can use the quasilinear approximation and express $\tilde{j}_\parallel \sim \tilde{E}_\parallel$ in terms of $\tilde{\varphi}$ and then in terms of the plasma displacement $\xi = (ck_y/B\omega)\tilde{\varphi}$. As a result, we find

$$\Pi = \hat{\nu}\frac{d}{dx}\Delta_\perp\varphi, \tag{10}$$

where $\hat{\nu}$ is the kinematic turbulent viscosity, which can be expressed in terms of the displacement ξ as follows:

$$\hat{\nu} = \frac{c^4\omega_{Bi}^2}{8\pi\omega_{pi}^2}\sum_{k,\,\omega}\frac{k_\parallel^2 k_\perp^4\eta}{\omega^2}\left(\frac{\partial^2}{\partial k_x^2}|\xi|^2 - \left|\frac{\partial\xi}{\partial k_x}\right|^2 - k_\perp^{-4}\left|\frac{\partial}{\partial k_x}k_\perp^2\xi\right|^2\right), \tag{11}$$
$$k_x = \nabla\langle\Phi\rangle/|\nabla\langle\Phi\rangle|.$$

Comparing this viscosity coefficient with the electron thermal diffusivity, we find

$$\hat{\nu} = \frac{m}{M} \chi_e \tag{12}$$

The vortex equation then becomes

$$\rho_0 \frac{d\Gamma}{dt} = (\mathbf{B}, \nabla) j + \nabla \hat{\mu} \nabla \Gamma, \tag{13}$$

where $\hat{\mu} = \rho_0 \hat{\nu}$. The last term in (13) describes a diffusion of the vorticity due to a small-scale $[k \sim (\omega_{pe}/c)]$ turbulence. The corresponding rotation decay time τ_φ is related to the plasma lifetime τ_{Ee} by

$$\tau_\varphi = \tau_{Ee} \beta \frac{M}{m}. \tag{14}$$

For the PDX device we find the estimate $\tau_\varphi^{\text{theo}} \approx 100$ ms, which agrees with the experimental value $\tau_\varphi^{\text{expt}} \approx 80\text{–}100\,\mu\text{s}$ (Ref. 2). For the PLT device we would have $\tau_\varphi^{\text{theo}} \approx 80$ ms in comparison with[1] $\tau_\varphi^{\text{expt}} \approx 20$ ms. In view of the approximate nature of expression (14), we judge the agreement to be satisfactory in this case also.

An interesting aspect of expression (14) is the strong density dependence $(\sim n^2)$. The decay time actually depends on the self-consistent profiles of the density, velocity, and temperature, which may cancel out the $\sim n^2$ dependence.

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