Exchange spin waves in a nonequilibrium system of oriented spins

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It is shown that undamped collective spin excitations, whose frequency is close to the exchange frequency, exist in a nonequilibrium system of oriented free-electron spins at $T=0$.

The discovery of optical spin orientation in semiconductors and spin injection with tunneling from a ferromagnet into a normal metal (see Refs. 1–4 and the references cited there) stimulated interest in the study of different effects in nonequilibrium systems of polarized spins. Aronov[5] predicted the existence of long-lived spin excitations with a quadratic dispersion law Bq^2 for small values of q in such systems. In this letter we show that the spin excitation spectrum has, in addition to this nonactivation branch, high-frequency exchange branches, which are separated from the first branch by a gap whose magnitude at $q=0$ is on the order of $\Omega_{\rm ex} = JR/\hbar$, where J is the exchange-interaction parameter, and $R = (N_\uparrow - N_\downarrow)/N$ is the degree of polarization of electrons which is determined by external effects (optical pumping, injection of polarized particles). These branches arise because of the dependence of the exchange interaction on the particle momenta.

The spectrum of excitations related to fluctuations of the spin density is determined by the poles of the dynamic magnetic susceptibility $\chi(\omega,\mathbf{q})$. To find $\chi(\omega,\mathbf{q})$ we shall use the Hartree-Fock approximation, in which allowance for the exchange interaction leads to a dependence of the energy of electrons with a definite projection of the spin α along the axis of quantization on their distribution function $n_{\mathbf{k}\alpha}$:

$$E_{\mathbf{k}\alpha} = \epsilon_{\mathbf{k}} - \sum_{\mathbf{k}'} V_{\mathbf{k}-\mathbf{k}'} n_{\mathbf{k}'\alpha}, \tag{1}$$

where $V_{\mathbf{k}-\mathbf{k}'}$ are the Fourier components of the screened Coulomb potential, and $\epsilon_{\mathbf{k}} = \hbar^2 k^2/2m$ is the unperturbed energy in the parabolic conduction band model.

In the simplest case of zero temperature $V_{\mathbf{k}-\mathbf{k}'}$ it can be replaced by the value at the Fermi surface, i.e., we can set $|\mathbf{k}| = |\mathbf{k}'| = k_F$. $V_{\mathbf{k}-\mathbf{k}'}$ will then depend only on the angle θ between $\mathbf{k}$ and $\mathbf{k}'$, and it can be expanded in a series of Legendre polynomials

$$V_{\mathbf{k}-\mathbf{k}'} = D^{-1}(\epsilon_F) \sum_l (2l+1) V_l P_l(\cos\theta), \tag{2}$$

where

$$V_l = \frac{2\pi e^2}{k_F^2} D(\epsilon_F) Q_l\left(1 + \frac{\kappa^2}{2k_F^2}\right), \tag{3}$$

$D(\epsilon_F)$ is the state density at the Fermi level, $Q_l(x)$ is the Legendre function of the second kind, e is the electron charge, and κ is the inverse screening radius. We note that only the first term in expansion (2) was taken into account in Ref. 5. In this approximation there are no exchange spin waves.

According to (1) and (2), the "exchange" gap in the particle spectrum is

$$E_{\mathbf{k}\downarrow} - E_{\mathbf{k}\uparrow} = JR = R\frac{e^2 k_F}{3\pi} \ln\left(1 + \frac{4k_F^2}{\kappa^2}\right). \tag{4}$$

It is related to the difference of the Fermi quasilevels of electrons with opposite spins $\delta\epsilon_F = \epsilon_{F\uparrow} - \epsilon_{F\downarrow}$ by the relation

$$\delta\epsilon_F = \frac{2}{3}R\,\epsilon_F - \hbar\Omega_{ex} \tag{5}$$

which is valid if the energy relaxation of electrons occurs more rapidly than the spin relaxation, since it is only in this case that the Fermi quasilevels have any meaning at all. Because of the absence of spontaneous spin ordering in this system, we shall assume below that the condition $JD(\epsilon_F)/N < 1$, which is the inverse of the well-known condition for the appearance of Stoner band ferromagnetism, holds. It thus follows from (5) that sign $\delta\epsilon_F$ = sign R.

The circular Fourier components of the transverse susceptibility $\chi^{\pm}$ are expressed in terms of the electron correlation functions $\delta\nu_{\mathbf{k}}^{\pm}(\omega,\mathbf{q}) = \langle a_{\mathbf{k}-\mathbf{q}\alpha}^{+} a_{\mathbf{k}-\alpha}\rangle$ (the upper sign corresponds to $\alpha = \uparrow$, and the lower sign corresponds to $\alpha = \downarrow$) in the form

$$\chi^{\pm}(\omega,\mathbf{q}) = -(g\mu_B/H_0^{\pm})\sum_{\mathbf{k}} \delta\nu_{\mathbf{k}}^{\pm}(\omega,\mathbf{q}), \tag{6}$$

where g is the spin gyromagnetic ratio, and μ_B is the Bohr magneton. Linearizing the starting equation of motion for the operator $a_{\mathbf{k}-q\alpha}^{+} a_{\mathbf{k}-\alpha}$ with respect to the amplitude of the perturbing magnetic field $H_0^{\pm} = H_{ox} \pm iH_{oy}$, we find, after substituting (1), (4), and (5), an integral equation for $\delta\nu_{\mathbf{k}}^{\pm}(\omega,\mathbf{q})$:

$$(\omega \mp \Omega_{ex} - \mathbf{q}\nabla_{\mathbf{k}}\epsilon_{\mathbf{k}} + i0)\,\delta\nu_{\mathbf{k}}^{\pm} - \left(\mathbf{q}\nabla_{\mathbf{k}}\epsilon_{\mathbf{k}} \pm \frac{\Omega_{ex}}{V_0}\right)\frac{\partial n_{\mathbf{k}}}{\partial\epsilon_{\mathbf{k}}}\sum_{\mathbf{k}'} V_{\mathbf{k}-\mathbf{k}'}\,\delta\nu_{\mathbf{k}'}^{\pm}$$

$$= -\frac{1}{2} g\mu_B H_0^{\pm}\left(\mathbf{q}\nabla_{\mathbf{k}}\epsilon_{\mathbf{k}} \pm \frac{\Omega_{ex}}{V_0}\right)\frac{\partial n_{\mathbf{k}}}{\partial\epsilon_{\mathbf{k}}}\ . \tag{7}$$

Let us transform this equation to a system of linear algebraic equations for

$$X_{lm}^{\pm}(\omega,\mathbf{q}) = g\mu_B\sum_{\mathbf{k}} Y_{lm}^{*}(\theta_{\mathbf{k}},\varphi_{\mathbf{k}})\,\delta\nu_{\mathbf{k}}^{\pm}(\omega,\mathbf{q}),$$

where $Y_{lm}(\theta_{\mathbf{k}},\varphi_{\mathbf{k}})$ are the spherical harmonics, and the magnetization is $M^{\pm} = -(4\pi)^{1/2} X_{00}^{\pm}$. The condition for the existence of a solution to the system of corresponding homogeneous equations gives an equation for determining the susceptibility poles. The selection of physically admissible solutions of this dispersion equation is based on the considerations developed in Ref. 5, according to which the excitation of spin waves is energetically favorable, in the nonequilibrium system of oriented spins, and their frequency has the opposite sign to that of an equilibrium ferromagnet studied in Ref. 6. As a result, we find that for $R > 0$ the spectrum of spin excitations consists of a clockwise-polarized branch $\omega_0^+(q)$, which begins at $q = 0$ with zero frequency, and a set of counterclockwise-polarized high-frequency ($l = 1,2, ...$) branches $\omega_l^-(q)$, beginning at

$$\omega_l^-(0) = -(1 - V_l/V_0)\Omega_{ex} . \tag{8}$$

At $R < 0$ the nonactivation branch is counterclockwise-polarized, while the high-frequency branches is clockwise-polarized. At $R > 0$ the spin waves can propagate only in the regions $\omega^{\pm} > \pm \Omega_{ex} + qv_F$ and $\omega^{\pm} < \pm \Omega_{ex} - qv_F$, in which there is no collisionless Landau damping. Analysis shows that since the frequencies of exchange waves increase with q, the branches corresponding to large values of l fall into the damped region at very small values of q.

We shall illustrate the nature of the dispersion dependences obtained by examining a simple case in which only the first two terms in expansion (2) need be considered, as done in the analysis of zero-sound in the theory of a Fermi liquid. For small $q(qv_F \ll \Omega_{ex})$ the dispersion equations for the waves of two different polarizations are

$$1 - V_0 + V_0(s^+ + u^+)Q_0(s^+) = 0, \tag{9}$$

$$1 - V_1 + 3V_1(s^- + u^-)s^- Q_1(s^-) = 0, \tag{10}$$

where $s^{\pm} = (\omega \mp \Omega_{ex})/qv_F$, and $u^{\pm} = \pm \Omega_{ex}/V_0 qv_F$. Expanding the Legendre functions in a series in powers of $1/s$ and then solving these equations by iterations with respect to qv_F/Ω_{ex}, we find for $R > 0$

$$\omega_0^+(q) = -(1 - V_0)\frac{(qv_F)^2}{3\Omega_{ex}} , \tag{11}$$

$$\omega_1^-(q) = -\left(1 - \frac{V_1}{V_0}\right)\Omega_{ex} + \frac{3}{5}(1 - V_1)\frac{V_0}{V_1}\frac{(qv_F)^2}{\Omega_{ex}} . \tag{12}$$

Equation (12) describes the dispersion of the exchange spin wave, and expression (11) in the limit $V_0 \ll 1$ coincides with the dispersion law for the low-frequency wave, obtained in Ref. 5 from another equation with the ferromagnetic sign for the exchange interaction.

One possible method for observing exchange waves is to investigate Raman scattering of linearly polarized light, which does not create additional spin unbalance. The correlation function of the magnetization fluctuations, which is determined by the imaginary part of $\chi^{\pm}$, has in the region of collisionless damping sharp peaks at frequencies which coincide with the frequencies of exchange spin waves. Because of this circumstance, the scattered-radiation spectrum exhibits peaks which are displaced relative to the frequency of the incident wave by an amount on the order of Ω_{ex}.

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