Scattering of electromagnetic radiation by a nonequilibrium Josephson junction

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(Submitted 21 June 1984)
Pis'ma Zh. Eksp. Teor. Fiz. **40**, No. 4, 134–136 (25 August 1984)

Two additional harmonics (satellites) are predicted to appear in an electromagnetic wave as it is scattered by a nonequilibrium Josephson junction. The effect stems from oscillations of the density of electronic excitations and lies within the capabilities of spectroscopic measurements. The optimum conditions for corresponding experiments are found.

1. The deviation from equilibrium in a Josephson junction is usually slight, and in the most favorable case of a junction made from thin films this deviation is determined by the parameter $\nu/\gamma \ll 1$, where ν is the probability for single-particle tunneling, and γ is the index of the energy decay of the nonequilibrium excitations. Even if the deviation from equilibrium is only slight, however, there are some interesting and observable effects.

Let us examine the kinetic equation

$$u_\epsilon \dot{n}_\epsilon = Q(n_\epsilon) + J(n_\epsilon), \tag{1}$$

which describes the behavior of the nonequilibrium distribution function of the excitations, n_ϵ, in one of the films of a junction (see the inset in Fig. 2). Here $Q(n_\epsilon)$ is the tunneling source of the deviation from equilibrium ($Q \sim \nu$), $J(n_\epsilon)$ is the inelastic-collision integral ($J \sim \gamma$) $u_\epsilon = \epsilon\theta(\epsilon^2 - \Delta^2)/\sqrt{\epsilon^2 - \Delta^2}$, and Δ is the band gap in the film. The source $Q(n_\epsilon)$ is described by[1)]

$$Q(n_\epsilon) = Q^0(n_\epsilon) + Q^1(n_\epsilon)\sin 2Vt + Q^2(n_\epsilon)\cos 2Vt, \tag{2}$$

where V is the voltage across the junction ($e = \hbar = 1$). The presence of oscillations in (2), which are associated with the macroscopic phase coherence in the superconductors, warrants a special comment.

It is usually assumed that for single-particle excitations the role played by the nondiagonal long-range order in superconductors reduces to giving rise to coherence factors (in the matrix elements for transitions from one state to another) and to a singularity in the density of electronic levels. It follows from (1) and (2) that in nonequilibrium superconductors the distribution function of the excitations may also depend directly on the coherent phase difference. We are accordingly interested in experiments which might reveal effects of this type.

2. We consider the scattering of weak electromagnetic radiation at a frequency $\omega_0 < 2\Delta$ by a nonequilibrium Josephson junction. We use a collision operator representing collisions of the photon field with electrons. If $\omega_0 < 2\Delta$ and $N_{\omega_0} \gg 1$ (N_{ω_0} are the occupation numbers of photons of frequency ω_0), this operator is

$$\hat{\Phi} = N \iint_{\Delta}^{\infty} d\epsilon d\epsilon' \delta(\epsilon - \epsilon' - \omega_0)(u_\epsilon u_{\epsilon'} + v_\epsilon v_{\epsilon'})(n_\epsilon - n_{\epsilon'}). \quad (3)$$

Here n_ϵ is the distribution function of the electronic excitations, and $v_\epsilon = \Delta u_\epsilon / \epsilon$. We do not need the explicit expression for the factor $N (N \sim N_{\omega_0})$. We can use (3) to calculate the absorption coefficient for a transmitted wave. Assuming a temperature $T \sim T_c$, we distinguish a small deviation δn_ϵ of the excitation distribution function $n_\epsilon = n_\epsilon^0 + \delta n_\epsilon$ from the equilibrium function $n_\epsilon^0 = [\exp(|\epsilon|/T) + 1]^{-1}$, and we expand it in a Fourier series:

$$\delta n_\epsilon = \delta n_\epsilon^0 + \delta n_\epsilon^1 \sin 2Vt + \delta n_\epsilon^2 \cos 2Vt. \quad (4)$$

Here we are ignoring the higher-order harmonics which are quadratic in ν/γ. To find the Fourier amplitudes, we use (1). Since ν/γ is small, we can assume that the distribution functions in (2) are equilibrium functions; we then find

$$Q^0(\epsilon) \approx \nu [u_\epsilon u_{\epsilon - V} (n_{\epsilon - V}^0 - n_\epsilon^0) - u_\epsilon u_{\epsilon + V} (n_\epsilon^0 - n_{\epsilon + V}^0)],$$

$$Q^1(\epsilon) \approx \nu [v_\epsilon w_{\epsilon - V} (n_\epsilon^0 - 1/2) - v_\epsilon w_{\epsilon + V} (n_\epsilon^0 - 1/2)],$$

$$Q^2(\epsilon) \approx \nu [v_\epsilon v_{\epsilon - V} (n_\epsilon^0 - n_{\epsilon - V}^0) - v_\epsilon v_{\epsilon + V} (n_{\epsilon + V}^0 - n_\epsilon^0)]. \quad (5)$$

Here $\epsilon \geqslant \Delta$; $w_\epsilon = \Delta \theta (\epsilon^2 - \Delta^2)/\sqrt{\Delta^2 - \epsilon^2}$; $\nu^{-1} = 4N(0)RSd$, where R is the resistance of the insulating layer of the junction; d is the thickness of the superconducting film; S is its area; Δ' is the band gap in the second film—the injector—to which the functions in (5) with the displaced arguments apply; and $N(0)$ is the density of electronic levels at $T > T_c$. In (5), as in (3), we ignore the unbalance of the branches, since it is a small quantity of higher order. Assuming $T \sim T_c$, at which there are many equilibrium excitations in the film, we use the relaxation-time approximation for the collision integral $(\epsilon \geqslant \Delta)$ in (1):

$$J(n_\epsilon) \approx -\gamma \frac{\epsilon}{\sqrt{\epsilon^2 - \Delta^2}} \delta n_\epsilon. \quad (6)$$

The steady-state correction to n_ϵ^0 is then determined by the term Q^0 in (5), which reduces to an inconsequential renormalization of n_ϵ^0. Substituting (4)–(6) into (1) and (2), we find the amplitudes in (4):

$$\delta n_\epsilon^1 \approx \frac{Q_2(\epsilon)\omega_J + Q_1(\epsilon)\gamma}{u_\epsilon(\omega_J^2 + \gamma^2)}; \quad \delta n_\epsilon^2 \approx \frac{Q_2(\epsilon)\omega_J - Q_1(\epsilon)\gamma}{u_\epsilon(\omega_J^2 + \gamma^2)}; \ \omega_J = 2V. \quad (7)$$

Substituting (7) into (4) and then into (3), we find

$$\hat{\Phi} = \Phi_0 + \Phi_1 \sin \omega_J t + \Phi_2 \cos \omega_J t, \quad (8)$$

where the factors Φ^i are easily found. The absorption coefficient for electromagnetic radiation is determined by (8) and thus oscillates over time. This oscillation causes a modulation of a transmitted wave and gives rise to satellites with frequencies $\omega_0 \pm \omega_J$.

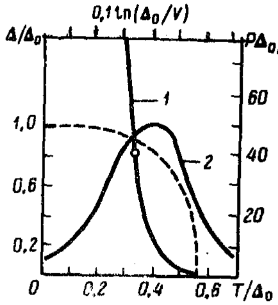

FIG. 1. The transformation coefficient P in a lead film. 1—As a function of the temperature $T(\omega_0\sim 300$ GHz, $V\sim 20\,\mu$V, $\gamma\sim 10$ GHz, $\Delta=\Delta'$) (the circle shows the temperature of the helium bath, $T=4.2$ K; 2—as a function of the voltage V ($\omega_0\sim 300$ GHz, $T=4.2$ K, $\gamma\sim 10$ GHz, $\Delta=\Delta'$); dashed line —plot of the equilibrium band gap $\Delta(T)$ in the BCS model. $\Delta_0=\Delta(T=0)$.

3. To estimate the magnitude of the effect, we introduce the "transformation" coefficient P, which is the ratio of the intensity of a satellite to the intensity of the absorbed wave at the fundamental frequency ω_0. It then follows from (8) that

$$P\sim\sqrt{\Phi_1^2+\Phi_2^2}\,/\,2\Phi_0\,. \tag{9}$$

Figures 1 and 2 show functions $P(V,\,T,\,\omega_0,\,\Delta',\gamma)$ found numerically for a lead film. Similar results were found for other metals (niobium, tantalum, etc.). The in-

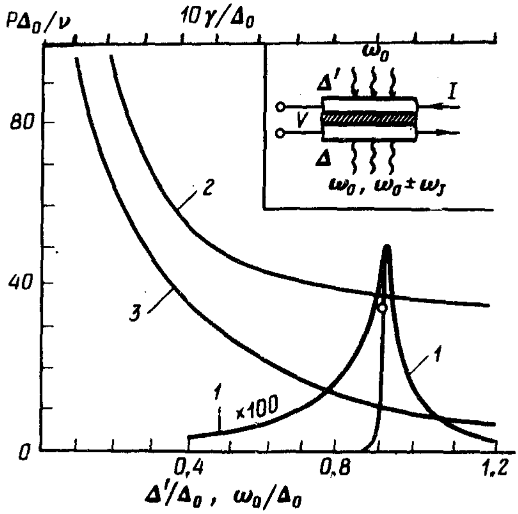

FIG. 2. 1—P as a function of the injector gap $\Delta'(\omega_0\sim 300$ GHz, $T=4.2$ K, $V\sim 20\,\mu$V) (the circle shows the case $\Delta=\Delta'$); 2—P as a function of the frequency of the incident radiation, ω_0 ($\gamma\sim 10$ GHz, $V\sim 40\,\mu$V, $T\sim 4.2$ K, $\Delta=\Delta'$); 3—P as a function of the decay index $\gamma(\omega_0\sim 300$ GHz, $T=4.2$ K, $V\sim 20\,\mu$V, $\Delta=\Delta'$).

crease in P with decreasing temperature (Fig. 1) results from a decrease in the number of equilibrium excitations and an effective "brightening" of the film [for clarity, Fig. 1 shows the function $\Delta(T)$ according to the BCS model). A plot of P versus the voltage across the function, V, has a maximum near $V \sim \gamma$. The coefficient P does not depend on the frequency of the probing electromagnetic radiation, ω_0, if this frequency is high enough ($\gtrsim 200$ GHz), but at lower frequencies it increases sharply. This effect is ultimately due to a singularity in the density of electronic levels which is peculiar to superconductors (in the calculations, this diverging singularity was cut off at energies $\epsilon \sim \gamma \sim 10$ GHz). The coefficient P behaves in a curious way as a function of the injector band gap Δ' (curve 1 in Fig. 2): At Δ' 1% below Δ, P has decreased by an order of magnitude. At Δ' 1% above Δ, on the other hand, P has increased significantly. This behavior may prove useful in detecting the satellites: In an asymmetric junction, oscillations of the density of excitations are manifested only in the film with the smaller gap. For a Pb film with $d \sim 10^3$ Å and $RS \sim 10^{-5} \Omega\,\text{cm}^2$, we estimate $\nu/\Delta_0 \sim 2.6 \times 10^{-5}$ and $P \sim 10^{-3}$ at $T = 4.2$ K. The detection of an effect of this order is completely feasible by spectroscopic methods.

We might note that an analogous effect should occur in the scattering of sound by a junction: the appearance of satellites in the transmitted wave. This effect obviously could not be attributed to oscillations in the Josephson current.

The observation of satellites with frequencies $\omega_0 \pm \omega_J$ and amplitudes in accordance with Figs. 1 and 2 (i.e., an oscillation of the density of electronic excitations) would clearly be of interest.

[1]A detailed derivation will be reported elsewhere. Result (2) is derived in the same approximation in the parameter ν/γ as the Josephson effect itself, so that (2) is not a consequence of the Josephson effect.

Translated by Dave Parsons
Edited by S. J. Amoretty