The axial-anomaly puzzle in supersymmetry gauge theories

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It is shown in $n = 1$ supersymmetry gauge theories that it is not possible to introduce an axial current which would be a superpartner of the energy-momentum tensor and which would belong to the same supermultiplet as the latter. The arguments are based on extremely weak assumptions: the Adler-Bardeen theorem in two loops and the existence of a Wess-Zumino supergauge.

The anomalies in supersymmetry gauge theories have recently attracted much attention.[1] Classically, there exists a supermultiplet of conserved currents which includes an axial current a_μ, a supercurrent $S_{\mu\alpha}$ ($\alpha = 1,2$), and an energy-momentum tensor $\Theta_{\mu\nu}$ (Ref. 2). The quantum-mechanical corrections induce anomalies in $\partial_\mu a_\mu$, $\gamma_\mu S_{\mu\alpha}$, and $\Theta_{\mu\mu}$. The standard assumption is that these corrections also form a supermultiplet.

We will focus on the particular case of a purely gauge supersymmetry theory ($n = 1$) with an SU(N) gauge group. The equation for an anomaly is then

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$$D^{\alpha} J_{\alpha\dot{\alpha}} = \frac{1}{3}\frac{\beta(\alpha_s)'}{\alpha_s}\bar{D}_{\dot{\alpha}} \operatorname{tr} \bar{W}^2 ,$$
$$J_{\alpha\dot{\alpha}} = \operatorname{tr}(W_{\alpha} e^{-gV} \bar{W}_{\dot{\alpha}} e^{gV}), \qquad \alpha_s = g^2/4\pi . \tag{1}$$

Here $\beta(\alpha_s)$ is the Gell-Mann–Low function, D^{α} is a spinor derivative, V is a superfield which generalizes the concept of vector potential, and W_{α} is a superstress, which incorporates the gluino field and the stress tensor of the gluon field:

$$W_{\alpha} = \lambda_{\alpha} + G_{\alpha\beta}\,\theta^{\beta} + D\theta_{\alpha} + D_{\alpha\dot{\beta}}\bar{\lambda}^{\dot{\beta}}\,\theta^2$$

(D is an auxiliary field without a kinetic term).

The problem is that, according to the Adler-Bardeen theorem,[3] the divergence of the axial current contains only the first order in g^2, while the conformal anomaly is proportional to the β function and thus generally incorporates all orders in g^2. Supersymmetry requires that $\partial_{\mu} a_{\mu}$ also be proportional to the β function. How do we reconcile this requirement of supersymmetry with the explicit result?

Another aspect of the problem is the analysis of the anomalous dimensionality of a_{μ} (Ref. 4). Since a_{μ} is not conserved, there is no reason to assume that its anomalous dimensionality is zero. Furthermore, an explicit calculation indicates the opposite: The two-loop anomalous dimensionality is nonzero.[5] On the other hand, supersymmetry puts a_{μ} in the same multiplet as the tensor $\Theta_{\mu\nu}$, which obviously has a zero dimensionality.

It is generally believed that the way out of this paradox is the existence of two axial currents. One, denoted by a_{μ}^{AB}, satisfies the Adler-Bardeen theorem, while the other a_{μ}^{SS}, belongs along with $S_{\mu\alpha}$ and $\Theta_{\mu\nu}$ to a supermultiplet $J_{\alpha\dot{\alpha}}$. This scheme was formulated most clearly and most comprehensively in a recent paper by Grisaru and West.[6]

In the present letter we demonstrate that if an Adler-Bardeen current a_{μ}^{AB} does exist, then a supersymmetry current a_{μ}^{SS} cannot be determined. By a_{μ}^{SS} we mean a current which satisfies two requirements: a) $\partial_{\mu} a_{\mu}^{SS} \sim \beta(\alpha_S)/\alpha_S$ in accordance with (1); b) the anomalous dimensionality of a_{μ}^{SS} is zero (see the discussion above). The proof of this assertion is based on an observation by Grisaru and West,[6] according to which the difference between a_{μ}^{SS} and a_{μ}^{AB} must reduce to a subtractive constant, which may arise from a difference in renormalization procedures. Our logic here is that there exist two independent limitations on the constant [requirements a) and b) above], and these limitations are incompatible.

We begin with the assumption that an Adler-Bardeen current with the following properties is defined in the theory:

$$\partial_{\mu} a_{\mu}^{AB} = \frac{N\alpha_s}{4\pi} G_{\mu\nu}^{a} \tilde{G}_{\mu\nu}^{a},$$
$$a_{\mu}^{AB} = \frac{1}{2}\bar{\lambda}^{a}\gamma_{\mu}\gamma_5\lambda^{a}, \tag{2}$$

where N is the number of loops, and $\widetilde{G}^a_{\mu\nu} = (1/2)\epsilon_{\mu\nu\alpha\beta}G^a_{\alpha\beta}$ is a dual tensor. Although the Adler-Bardeen theorem applies in all orders in g^2, the only point of importance for our purposes is that (2) does not contain $(g^2)^2$ terms. Furthermore, in two loops there is no need to fix the definition of the β function, since the first two coefficients in it do not depend on the definition and are given by[8]

$$\beta(\alpha_s) = -\frac{3N\alpha_s^2}{2\pi}\left(1 + \frac{N\alpha_s}{2\pi}\right). \tag{3}$$

Obviously, a_μ^{SS} and a_μ^{AB} coincide in the classical case. This coincidence means in turn that the coefficient $O(\alpha_s)$ in $\partial_\mu a_\mu^{SS}$ and $\partial_\mu a_\mu^{AB}$ are the same, since they are determined unambiguously by the imaginary part of the corresponding triangle diagram. In order α_s, the difference between the two different axial currents reduces to a subtractive constant. Furthermore, in the Wess-Zumino supergauge[7] the only subtractive constant with the appropriate dimensionality is the term $C\bar{\lambda}^a\gamma_\mu\gamma_5\lambda^a$. The appearance of a subtractive constant of this type with a coefficient $O(\alpha_s)$ would be manifested as a divergence of $\partial_\mu a_\mu$ in order α_s^2.

We fix the subtractive constant in such a manner that the divergence of the supersymmetry current a_μ^{SS} is proportional to the Gell-Mann–Low function β. It follows from (3) that

$$a_\mu^{SS} = a_\mu^{AB} + \frac{N\alpha_s}{2\pi}\frac{1}{2}\bar{\lambda}^a\gamma_\mu\gamma_5\lambda^a. \tag{4}$$

The next step is to calculate the anomalous dimensionality of this current. As was mentioned above, the anomalous dimensionality of the Adler-Bardeen current is not zero,[5] and it is associated with a two-loop diagram, in which the axial current converts into two gluons, which then convert into two gluinos. The result of the calculation is conveniently presented as a renormalization-invariant combination:

$$a_\mu^{AB}(\mu)\left[1 - \frac{N\alpha_s(\mu)}{2\pi}\right] = \text{renorm-invariant}. \tag{5}$$

Our result for the anomalous dimensionality of a_μ^{AB} agrees with the result derived by Crewther,[5] but it differs in sign from the corresponding expression derived by Espriu and Tarrach.[9]

Returning to the supersymmetry axial current, we rewrite (4) as

$$a_\mu^{SS} = \left[1 + \frac{N\alpha_s(\mu)}{2\pi}\right]a_\mu^{AB}(\mu). \tag{6}$$

If we identify this current with that term of the same multiplet which contains $\Theta_{\mu\nu}$, we must assume that its anomalous dimensionality is zero. This assumption, however, is in direct contradiction of Eq. (5). This completes the proof of the fact that a supersymmetry axial current does not exist if the Adler-Bardeen current does exist.

Since this derivation is radical, it is worthwhile reviewing the assumptions on which it is based: a) There exists a current which satisfies the Adler-Bardeen theorem in two loops; b) we have supergauge invariance of the supermultiplet of currents (i.e., we can work systematically in the Wess-Zumino supergauge).

A contradiction has been observed in the $n = 1$ gauge theories without matter, and it is quite possible that a radical change in the theory will be required. Although formally, in the theory itself, the axial current is a foreign entity, the axial charge belongs to a set of operators which generate a superconformal group, which is in turn the symmetry group of the classical equations of motion. Furthermore, the classical symmetry plus the renormalizability of the theory fix the β function in this theory (see Ref. 4 for further details and further references).

Analysis of the anomaly in the external gravitational field suggests the direction in which the understanding of the theory should change (this case was brought to our attention by R. Kallosh; see Ref. 10). In this case a similar "paradox" arises even in the single-loop approximation, and the problem is accordingly simpler to analyze and interpret. Our understanding of the problem in this case is as follows: A standard calculation of the triangle diagram for an axial current with the help of the unitarity condition leads to a nonsupersymmetric answer. Specifically, the coefficients for the axial and conformal anomalies do not satisfy the condition of equality which follows from supersymmetry. The solution proposed by Kallosh[10] works from the basis that the scalar field and the field of the antisymmetric tensor generate different anomalies, although the two theories are equivalent on the mass shell. In our opinion, this solution implies a change in the imaginary part and thus a change in the low-energy content of the theory. We hope to return to this problem in a separate paper.

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