Superfield action for an $N = 1$ supergravity

V. G. Zima
A. M. Gor'kiĭ Khar'kov State University

(Submitted 11 March 1984; resubmitted 23 July 1984)
Pis'ma Zh. Eksp. Teor. Fiz. **40**, No. 4, 168–170 (25 August 1984)

A new formulation of an $N = 1$ supergravity with an auxiliary superfield is proposed. This formulation incorporates all known formulations as particular cases.

The basic entities in a superfield description of an $N = 1$ supergravity are (1) a real supermanifold $\Omega^{4,4}$, which is locally parametrized by the four ordinary coordinates x^m and four anticommuting coordinates θ^μ, $\bar{\theta}_{\dot{\mu}}$ (denoted collectively as z^M), which we call the "physical superspace," and (2) a connection given on $\Omega^{4,4}$ with values in the algebra of the spinor expansion of the Poincaré group. If the chiral superfields of the global supersymmetry are to have the corresponding local analogs, type 1 constraints[1)] $T^c_{\alpha\beta} = T^{\dot{\gamma}}_{\alpha\beta} = 0$, must be imposed on the connection.[1] To eliminate from consideration the unessential superfields, we impose the conventional constraints, whose choice is governed by considerations of convenience, say, $T^c_{\alpha\dot{\beta}} = 2i\sigma^c_{\alpha\dot{\beta}}$ (this condition is, of course,[2] not "purely" conventional), $T^{\gamma}_{\alpha\dot{\beta}} = T^c_{\alpha b}(\tilde{\Sigma}^b_c)^{\gamma}_{\dot{\beta}} = 0$, and $R_{\alpha\dot{\beta}} = 0$. A distinctive feature of the supergravity under consideration here is the type 3 constraint, by means

 0021-3640/84/160928-03$01.00

of which we exclude the extraneous superfields. For the minimal supergravity, this would be $T_\alpha \equiv T^b_{\alpha b} = 0$, and for a nonminimal supergravity it would be $\frac{1}{3}R^{\alpha\beta}_{\alpha\beta} - \bar{\zeta} D^\alpha T_\alpha - \bar{\zeta}^2 T^\alpha T_\alpha = 0$, where $\zeta = n + 1/3n + 1$, and n is the complex Siegel parameter.[3] The imposition of these constraints not only resolves these problems but also eliminates higher-order spins from the theory.

It is convenient to solve the constraints of type 1 (as is done in detail for the minimal supergravity in Ref. 4). In general, the solution of type 1 constraints gives rise to an approach which was suggested by, and worked out in detail by, Ogievetskiĭ *et al.*[5] In this approach, the physical superfield $\Omega^{4,4}$ is a hypersurface in a complex superfield $\mathbb{C}^{4,4} = \{z^M_L\} = \{x^m_L, \theta^\mu_L, \bar{\varphi}^L_{\dot\mu}\}$, which is found by deforming the real hyperplane $R^{4,4}$. The coordinates of the points $\Omega^{4,4}$ in the factorspace $\mathbb{C}^{4,4}/R^{4,4}$ are arbitrary functions of the real coordinates in $R^{4,4}$:$x^m_L = x^m + i\mathscr{H}^m(z)$, $\theta^\mu_L = \theta^\mu$,$\bar{\varphi}^L_{\dot\mu} = \bar{\theta}_{\dot\mu} + \bar{H}_{\dot\mu}(z)$. The superfields $\mathscr{H}^m$ (which is real) and H^μ, which determine the nesting of $\Omega^{4,4}$ in $\mathbb{C}^{4,4}$ serve as prepotentials. Acting in $\mathbb{C}^{4,4}$ is a group of analytic transformations of a "triangular" type which, by definition, leave invariant the chiral subspace $\mathbb{C}^{4,2} = \{z_{0L}^M\} = \{x^m_L, \theta^\mu_L\}$:$z'_{0L} = z_{0L} + \lambda(z_{0L})$,$\bar{\varphi}'_L = \bar{\varphi}_L + \bar{\rho}(z_L)$. These transformations induce on $\Omega^{4,4}$, as on a hypersurface, the group of supergravity: a conformal group if there are no restrictions or an Einstein group if we are restricted to a subgroup which satisfies a condition that can conveniently be written

$$I^\zeta \bar{\Lambda}^{-1} = 1. \tag{1}$$

Here $I = \mathrm{Ber}(\partial z'_{0L}/\partial z_{0L})$, and $\bar{\Lambda} = \det(\partial\bar{\varphi}'_L/\partial\bar{\varphi}_L)$. For nonreal ζ this condition can be written in the right-handed (conjugate) coordinates in terms of $\bar{\zeta} \neq \zeta$, so that the left- and right-handed coordinates are nonequivalent. This statement means that for nonreal ζ in a $N = 1$ supergravity we do not have P invariance or C invariance. On the other hand, we do have CP invariance.

Fixing the gauge with respect to local Lorentz transformations, we can write a spinor covariant derivative of the scalar superfields in the form $\nabla_\alpha = F\partial/\partial\varphi^\alpha_R$, where the superfield F, along with the nesting conditions, determines the coupling of the derivatives ∇_α with the ordinary ∂_M. Now the type 2 constraints generate reference coefficients and connection coefficients in terms of the nesting functions F and $\bar{F}$.

The type 3 constraints play a double role. First, they restrict the group of analytic triangular transformations of the superfield $\mathbb{C}^{4,4}$ to subgroup (1). Second, they express F and $\bar{F}$ in terms of the nesting functions.

It turns out that in postulating restriction (1) on the group in the $N = 1$ supergravity we can avoid the imposition of type 3 constraints. The superfields F and $\bar{F}$ become auxiliary fields. We write the action integral in the form

$$A(n) = \frac{1}{2\kappa^2}\left\{\int d^8 z\,\hat{c}^{-1}\,|F|^{-4} - \frac{n+1}{2n}\int d^8 z_L\,\bar{F}^{-(4n/(n+1))} - \frac{\bar{n}+1}{2\bar{n}}\int d^8 z_R\,F^{-(4\bar{n}/(\bar{n}+1))}\right\}, \tag{2}$$

where $\hat{c} = \mathrm{Ber}(\hat{\Delta}_A z^M)$, $\hat{\Delta}_a = (i/4)\tilde{\sigma}^{\dot\alpha\alpha}_a\{\hat{\Delta}_\alpha, \hat{\Delta}_{\dot\alpha}\}$, $\hat{\Delta}_\alpha = \partial/\partial\varphi^\alpha_R$. If Re$n \neq 0$, a variation of (2) with respect to F and $\bar{F}$ gives us equations which are equations of motion for these functions and which can be found by solving the type 3 constraints. Substituting the functions F and $\bar{F}$ found in this manner into (2), we find the ordinary action of a nonminimal $N=1$ supergravity of the type $-[(n+\bar{n})/(2\kappa^2|n|^2)]\int d^8zE(n)^6$ (Ref. 6) for the general case of complex n. This expression, in contrast with (2), has no singularity for $n=-1$. For $n=-1/3$ (the minimal supergravity), condition (1) does not restrict $\bar{\rho}(z_L)$, so that we can calibrate H^μ (we can nest $\Omega^{4,4}$ into $\mathbb{C}^{4,2}$). The transition to $n=0$ becomes trivial: In the limit (cf. Ref. 7), using $\int d^8z_L=0$, we find

$$A\ (0)=\frac{1}{2\kappa^2}\{\int d^8z\hat{c}^{-1}|F|^{-4}+\int d^8z_L\ln\bar{F}^2+\int d^8z_R\ln F^2\}. \tag{3}$$

For this action the condition $\mathrm{Ber}(\partial z_R/\partial z_L)=1$ is an equation of motion which corresponds to a variation with respect to the phase of F. Taking it and the equation for $|F|$ into account, we can rewrite action (3) in the standard form[8] $(1/\kappa^2)\int d^8zE(0)\ln|F|^2$.

Purely imaginary values of n play a special role. For them, as in the case $n=0$, a variation with respect to F and $\bar{F}$ allows us to find only $|\exp(\sqrt{(1-n)/(1+n)}\ln\bar{F})|$ under the condition $\sqrt{(1-\bar{n})/(1+\bar{n})}\ln\hat{r}=\sqrt{(1-n)/(1+n)}\ln\hat{l}$, where $\hat{r}=\mathrm{Ber}(\hat{\Delta}_A z_R^M)$. When we take this fact into account, we find that action (2) vanishes. Consequently, there exists no formulation of an $N=1$ supergravity in terms of the nesting functions for imaginary n: It is impossible to construct an invariant superspace. The reason for the appearance of an invariant, $\exp\{\sqrt{(1-\bar{n})/(1+\bar{n})}\ln\hat{r}-\sqrt{(1-n)/1+n)}\ln\hat{l}\}$, for these n is that the degrees of freedom, which are generally expended on constructing F, are now left "idle." We note that the type 3 constraints in the case of imaginary n determine F within Weyl transformations $\bar{F}\to\exp(i\sqrt{(1-n)/(1+n)}\varphi)\bar{F}$, where φ is real (this circumstance was pointed out in Ref. 4 for the case $\zeta=1/3$).

In cases with material superfields, and also upon quantization, the formulation proposed here may have some obvious advantages over those[3,5,6] in which the expression for F in terms of $\mathcal{H}^m$ and H^μ is postulated.

I wish to thank D. V. Volkov, V. I. Ogievetskiĭ, É. S. Sokachev, and A. S. Gal'perin for useful discussions.

[1)]It is convenient here to switch places of type 1 and and type 2 constraints in the classification of Gates *et al.*[1]

[1]S. J. Gates, K. S. Stelle, and P. C. West, Nucl. Phys. **B169**, 347 (1980).
[2]V. P. Akulov, D. V. Volkov, and V. A. Soroka, Teor. Mat. Fiz. **31**, 12 (1977).
[3]W. Siegel and S. J. Gates, Nucl. Phys. **B147**, 77 (1979).
[4]I. V. Frolov, Yad. Fiz. **36**, 1014 (1982) [Sov. J. Nucl. Phys. **36**, 593 (1982)].
[5]V. I. Ogievetsakiĭ and É. S. Sokoachev, Yad. Fiz. **31**, 205 (1980) [Sov. J. Nucl. Phys. **31**, 108 (1980)]; A. S. Gal'perin, V. I. Ogievetskiĭ and É. S. Sokachev, Pis'ma Zh. Eksp. Teor. Fiz. **35**, 263 (1982) [JETP Lett. **35**, 330 (1982)].
[6]S. J. Gates and W. Siegel, Nucl. Phys. **B263**, 519 (1980).
[7]A. A. Roslyĭ and A. S. Shvarts, Yad. Fiz. **37**, 786 (1983) [Sov. J. Nucl. Phys. **37**, 466 (1983)].
[8]A. S. Galperin, V. I. Ogievetskiĭ, and E. S. Sokatchev, Preprint E2-82-211, Joint Institute for Nuclear Research, Dubna, 1982.

Translated by Dave Parsons
Edited by S. J. Amoretty