Spin-wave spectrum of antiferromagnetic superconductors

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It is shown that the long-wavelength spin-wave spectrum of antiferromagnetic superconductors differs substantially from the usual spectrum. In the case of an easy-axis anisotropy, the spectrum should have a dip at a finite value of the wave vector.

1. Since the discovery of superconducting ternary compounds with a regular lattice of rare-earth (RE) atoms $RERh_4B_4$ and $REMo_6S_8$, attention has been focused primarily on the problem of the coexistence of superconductivity and ferromagnetism. In the coexistence phase there arises in this case an inhomogeneous magnetic structure, which after further cooling can transform into the normal ferromagnetic phase. This situation occurs in $ErRh_4B_4$ and $HoMo_6S_8$ (see, for example, the review in Ref. 1). However, most superconducting ternary compounds exhibit antiferromagnetic ordering at the Néel temperature $T_N < T_c$, where T_c is the temperature of the superconducting transition.[1,2]

Superconductivity and antiferromagnetism have virtually no effect on each other. In particular, superconductivity in no way changes the antiferromagnetic order, since the average value (at the superconducting correlation length $\xi_0 = 0.18 v_F/T_c$) of the exchange field and of the magnetization of the antiferromagnet is zero. The situation is different, however, for spin waves in antiferromagnetic superconductors (AS). When the localized moments in the spin wave tilt away from their equilibrium position, a ferromagnetic moment, which strongly affects the superconductivity, appears.

2. In determining the spin-wave spectrum of AS, we shall examine the case of a two-sublattice antiferromagnet with an easy-axis anisotropy. As follows from currently available neutron-scattering data, AS belong precisely to this class. In describing the spin wave, we shall change over from the magnetization vectors of the sublattices $\mathbf{M}_1$ and $\mathbf{M}_2$ to the ferromagnetic vector $\mathbf{m} = \mathbf{M}_1 + \mathbf{M}_2$ and to the vector $\mathbf{l} = \mathbf{M}_1 - \mathbf{M}_2 - (\mathbf{M}_1^0 - \mathbf{M}_2^0)$, which describes the tilting of the antiferromagnetism vector away from the equilibrium vector $\mathbf{L}^0 = \mathbf{M}_1^0 - \mathbf{M}_2^0$. The free-energy functional is $\mathscr{F} = \mathscr{F}_m(\mathbf{l}, \mathbf{m}) + \mathscr{F}_{\text{int}}(\mathbf{m})$ where $\mathscr{F}_m(\mathbf{l}, \mathbf{m})$ is the usual magnetic functional which describes the system in the absence of superconductivity, while $\mathscr{F}_{\text{int}}$ describes the interaction of the superconducting and magnetic subsystems and depends only on the vector $\mathbf{m}$ (since only the ferromagnetic moment interacts effectively with the superconductivity).

Since the specific nature of the spin-wave spectrum of AS is seen in the long-wavelength region $q \ll a^{-1}$, where a is the magnetic rigidity, equal in order of magnitude to the interatomic distance, we can use as $\mathscr{F}_m$ the usual phenomenological functional (see, for example, Ref. 3).

$$\mathcal{F}_m = \int \left[\frac{\delta}{2}\mathbf{m}^2 + \frac{1}{2}a^2\frac{\partial \mathbf{l}}{\partial x_i}\frac{\partial \mathbf{l}}{\partial x_j} + \frac{K\mathbf{l}^2}{2}\right]d\mathbf{r}. \qquad (1)$$

In (1) the anisotropy of the magnetic rigidity is ignored and $\mathcal{F}_m$, for clarity, is taken in its simplest form. Furthermore, in (1) it is assumed that $\mathbf{L}^0$ is oriented along the easy axis z, the vectors $\mathbf{m}$ and $\mathbf{l}$ are perpendicular to $\mathbf{L}^0$, and both the exchange and electromagnetic interactions contribute to the constant δ. The ratio of their characteristic energies is $\alpha \sim h_0^2 N(0)/\mu^2 n$, where h_0 is the exchange field at $T=0$, $N(0)$ is the electronic state density at the Fermi level, μ is the magnetic moment of the RE, and n is their concentration. The exchange contribution usually dominates and $T_N \sim h_0^2 N(0)$, but in AS the electromagnetic energy is $2\pi\mu^2 n \sim 1$ K because of the anomalously low value of the exchange integral and all characteristic energies are of the same order of magnitude: $2\pi\mu^2 n \sim h_0^2 N(0) \sim T_N \sim (0.5-2)$ K, i.e., $\alpha \sim 1$. With regard to $\mathcal{F}_{\text{int}}$, we note that the superconductivity screens the long-wavelength part of the electromagnetic and exchange interactions, having virtually no effect on the short-wavelength part, (with $q \sim a^{-1}$). For this reason $\mathcal{F}_{\text{int}}$ is the difference between the long-range interactions in the superconducting and normal phases[4]:

$$\mathcal{F}_{\text{int}} = \int \left(\frac{B^2}{8\pi} - \mathbf{B}\mathbf{m} + 2\pi\mathbf{m}^2\right)dr + \frac{1}{2}\int \Big[Q(\mathbf{r}-\mathbf{r}')A(\mathbf{r})A(\mathbf{r}')$$

$$- \alpha\frac{\chi_s(\mathbf{r}-\mathbf{r}') - \chi_n(\mathbf{r}-\mathbf{r}')}{\chi_n^0}\mathbf{m}(\mathbf{r})\mathbf{m}(\mathbf{r}')\Big]d\mathbf{r}\,d\mathbf{r}', \quad \mathbf{B} = \operatorname{rot}A, \qquad (2)$$

where B is the magnetic field created by the magnetization, $\chi_n(\mathbf{r})$ and $\chi_s(\mathbf{r})$ are the spin susceptibilities in the normal and superconducting states, $\chi_n^0 = \mu_B^2 N(0)$, and $Q(\mathbf{r})$ is the superconducting electromagnetic kernel. Eliminating $\mathbf{A}$ and $\mathbf{B}$ with the help of Maxwell's equations, we obtain the complete functional $\mathcal{F}$ in the Fourier representation

$$\mathcal{F} = \sum_{\mathbf{q}}\left\{\frac{q^2a^2}{2}|\mathbf{l}_{\mathbf{q}}|^2 + \frac{K}{2}|\mathbf{l}_{\mathbf{q}}|^2 + \frac{\delta}{2}\left[1 + \frac{4\pi Q(\mathbf{q})}{\mathbf{q}^2 + 4\pi Q(\mathbf{q})}\right.\right.$$

$$\left.\left. + \alpha\left(1 - \frac{\chi_s(\mathbf{q})}{\chi_n^0}\right)\right]|\mathbf{m}_{\mathbf{q}}|^2\right\}. \qquad (3)$$

At temperatures $T \ll T_c$, for a clean superconductor with $q \ll \xi_0^{-1}$ we have $Q = 1/4\pi\lambda_L^2$, where λ_L is the London penetration depth, while $\chi_s \approx 0$, and for $q \gg \xi_0^{-1}$ we have $Q = 0.75/(q\xi_0\eta\lambda_L^2)$ and $\chi_s(q)/\chi_n^0 = 1 - \pi/(2q\xi_0)$. Writing in a standard manner the equations of motion for $\mathbf{l}$ and $\mathbf{m}$ by making use of (3) and then solving them, we find (assuming that the wave is transverse) the spin-wave spectrum $\omega(q)$

$$\omega^2(q) = \gamma^2\delta\left[1 + \frac{4\pi Q(q)}{q^2 + 4\pi Q(q)} + \alpha\left(1 - \frac{\chi_s(q)}{\chi_n^0}\right)\right][K + q^2a^2], \qquad (4)$$

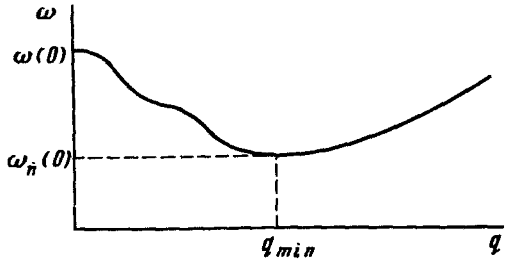

FIG. 1. Schematic representation of the spin-wave spectrum of an antiferromagnetic superconductor. The frequency of the antiferromagnetic resonance is $\omega(0) = \gamma[\delta(2+\alpha)K]^{1/2}$ and the wave vector corresponding to the minimum frequency is $q_{\min} \sim (K/a^2\xi_0)^{1/3}$.

where $\gamma = g\mu_B L_0/2$. The most significant change in the spectrum of the AS is the appearance of a strong — at the characteristic vectors $q \sim \lambda_L^{-1}$ and ξ_0^{-1} — q dependence of ω, leading to the appearance of a minimum in the curve $\omega(q)$ (see Fig. 1). The minimum in $\omega(q)$ occurs at $q_{\min} = (\pi\alpha K/4\xi_0 a^2)^{1/3} \ll a^{-1}$, while the frequency $\omega(q_{\min})$ is essentially the same as the frequency of the antiferromagnetic resonance $\omega_n(0) = \gamma(\delta K)^{1/2}$ in the normal phase.

3. The physical reason for the effect examined above is the higher "resistance" of the superconductor to the ferromagnetic moment. At the same time, as the wave vector increases, this resistance weakens rapidly. There are two characteristic scales: for $\lambda_L^{-1} \ll q \ll \xi_0^{-1}$, the electromagnetic rigidity is switched off, but the exchange rigidity remains, whereas for $q \gg \xi_0^{-1}$, the effect of superconductivity completely vanishes.

We note that this characteristic of the spin-wave spectrum of an AS is of a general nature and is not related to the simplified nature of the model examined. Studies of the spin-wave spectrum of AS can thus yield a wealth of information both on the magnetic properties of these compounds and on their superconducting characteristics. Compounds with higher Néel temperature — $NdRh_4B_4$ with $T_N \approx 1.3$ K, and also the alloys $Ho(Ir_x Rh_{1-x})_4B_4$ with $T_N \sim (2-3)$ K — seem to be more suitable for experimental studies.

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