Dimensional reduction as a source of stable mass in an expanded Yang-Mills theory

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A nontrivial dimensional reduction of an $N=1$, $d>4$ Yang-Mills theory which leads to a spontaneous symmetry breaking is accompanied by an activation of central charges in the algebra of the expanded supersymmetry in $d=4$, so that the associated masses of the supermultiplets do not acquire quantum corrections.

The incorporation of central charges in the algebra of an expanded supersymmetry is a systematic method for introducing masses in supersymmetry theories without increasing the dimensionality of the representation.[1] On the other hand, theories which are invariant under an expanded supersymmetry with central charges can be found by means of a nontrivial dimensional reduction.[2] Our purpose in this letter is to show that the masses obtained in this manner do not acquire quantum correction, i.e, are stable. Serious arguments for this possibility come from the results of explicit calculations of the single-loop corrections to the monopole mass in an $N=2$ Yang-Mills theory in $d=4$ (Ref. 3) and indirect arguments[4] based on an analysis of the dimensionality of the massive representations of the supersymmetry. We have proved that there are no quantum corrections to the mass in $d=2$ by explicitly carrying out a dimensional reduction by a diagram technique of the supersymmetry of the CP^{N-1} model.[5]

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We restrict the present letter to an analysis of the simple $d = 4$, $N = 2$ Yang-Mills system with the gauge group SU(2). This model is known to correspond to an $N = 1$ Yang-Mills system in $d = 6$, formulated in superspace in terms of superconnectednesses $A_V = (A_m, A_\alpha)$, which satisfy constraint conditions that can be resolved in terms of independent prepotentials $V_{ij}(x_m, \theta_{\alpha A}) i,j = 1, 2$ (Ref. 6). The representation $V_{ij} = \tilde{V}_{ij}(\mathbf{x},\mathbf{y},\theta^{\alpha i},\bar{\theta}^{\dot{\alpha}}_j)$ is convenient for the subsequent reduction $d = 4$, where $\theta^{\alpha 1}$ and $\bar{\theta}^{\dot{\alpha}}_j$ are Weyl spinors with $i, j = 1, 2$. In the noninteracting case, the A^{i}_{α} are expressed in terms of the $\tilde{V}_{ij}$ and the covariant supersymmetric derivatives $D_{\alpha i}$, $\bar{D}^{i}_{\dot{\alpha}}$ in closed form; in the $g \neq 0$ case, they can be expressed as a series in g. The trivial reduction $6 \to 4[\tilde{V}_{ij}(\mathbf{x}^{(4)},\mathbf{y}, \ldots) \equiv V_{ij}(\mathbf{x}^{(4)}, \ldots)]$ leads to a superfield formulation of the $N = 2$ Yang-Mills system in terms of $N = 2$ superfields. This description makes it possible to demonstrate the nonrenormalizability theorem by a background-field method and to show that the theory contains only a charge renormalization.[7]

We carry out the nontrivial reduction $\tilde{V}_{ij}(\mathbf{x},\mathbf{y}, \text{and} \ldots) \equiv U(y) V_{ij}(\mathbf{x}, \ldots) U^{-1}(\mathbf{y})$, where $U(\mathbf{y}) = \exp i y_k \mathscr{M}_k$, $[\mathscr{M}_1, \mathscr{M}_2] = 0$ (Ref. 2). In this spinor basis the covariant derivative is written $D^{(6)}_{\alpha i} = \partial / \partial \theta^{\alpha i} + i(\partial\!\!\!/ \bar{\theta})_{\alpha i} + \epsilon_{\alpha\beta} \epsilon_{ij} \theta^{\beta j} (\partial / \partial y_1 + i \partial / \partial y_2)$, where $\partial\!\!\!/ = \partial_\mu \sigma^\mu$. In our case, on the other hand, upon the trivial reduction $\partial / \partial y_i = 0$ and $D^{(6)}_{\alpha i} \to D^{(4)}_{\alpha i}$ the derivative $D^{(6)}_{\alpha i}$ becomes $\hat{D}_{\alpha i} = D^{(4)}_{\alpha i} + i \epsilon_{\alpha\beta} \epsilon_{ij} \theta^{\beta j} [(\mathscr{M}_1 - i \mathscr{M}_2), \ldots]$. The derivatives $\hat{D}_{\alpha j}$ and $\bar{D}^{j}_{\dot{\alpha}}$ satisfy an $N = 2$ supersymmetry algebra with central charges, which are realized as operators of an associated representation of the SU(2) algebra: $[\mathscr{M}_i, \ldots]$. Now replacing $D^{(4)}$ by $\hat{D}$ in the expression for the total action (which also includes ghost sectors),[7] we find a Yang-Mills theory which is invariant under the $N = 2$ supersymmetry with central charges. In this approach, all assertions regarding the renormalizability of the theory on the basis of the background-field method remain valid; i.e., there is a unique renormalization constant. We wish to emphasize that no additional renormalization of any sort is required because of the introduction of the new dimensional parameters $\mathscr{M}_i$, since $\mathscr{M}_i$ is present only through $\hat{D}$ in any of the expressions. As a result, the invariance of the renormalized action under the supersymmetry remains the same.

The meaning of the parameters $\mathscr{M}_i$ can easily be seen by performing a reduction in the quadratic form $\mathrm{Tr}\{1/4(\partial_m A_n(\mathbf{x},\mathbf{y}) - \partial_n A_m(\mathbf{x},\mathbf{y}))^2 + i/2 \cdot \bar{\psi}(\mathbf{x},\mathbf{y}) \check{\partial} \psi(\mathbf{x},\mathbf{y})$ of the physical fields in $d = 6$. Here we have $\partial^{(6)} \to (\partial^{(4)}, i[\mathscr{M}_1, \ldots], i[\mathscr{M}_2, \ldots])$, and the effect is to generate a spontaneous breaking of the SU(2) symmetry. As the result, we find a charged massive $N = 2$ vector supermultiplet with a mass $M^2 = |\mathscr{M}_1|^2 + |\mathscr{M}_2|^2$ and a dimensionless vector supermultiplet. As expected, the massive multiplet realizes a (complex) representation of the $N = 2$ supersymmetry with central charges.[1,8] This realization of the Higgs mechanism is equivalent to the choice of nonvanishing vacuum expectation values for the scalar fields along the planar directions of the indifferent potential $\mathrm{Tr}\{[A_4, A_5]\}^2$ of this model.[1] The Higgs mechanism can be implemented in a nonsupersymmetry theory in the same manner, but in the case of the $6 \to 4$ reduction for a Yang-Mills theory the massless boson fields (vectors + scalar) lead in $d = 4$ to nonremovable infrared singularities. In the supersymmetry case, on the other hand, it follows from our explicit analysis[9] in terms of $N = 1$ superfields that the Higgs phase is free of infrared singularities off the mass shell, so that both the diagram technique and the renormalization procedure are correct.

We thus see that the parameters of the supersymmetry transformations which have the meaning of masses are not renormalized. Furthermore, the renormalized effective action found by this diagram technique conserves the supersymmetry with the same values of the parameters $\mathscr{M}_i$ as for the seed; this is direct proof of the stability of the masses. A generalization of this reduction method to the case of a Yang-Mills system which is interacting with $N=2$ mass fields (hypermultiplets) leads to no fundamental difficulties. It should be noted that the central charges in this case can be realized in different ways on different $N=2$ supermultiplets and that their masses will generally be different. Although the reduction question for a $d=10$ Yang-Mills theory is more complicated, it can be expected that a corresponding result for it will also be found.

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